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ROYAL AERONAUTICAL SOCIETY 1866

Journal

of

THE ROYAL AERONAUTICAL SOCIETY

MARCH 1955

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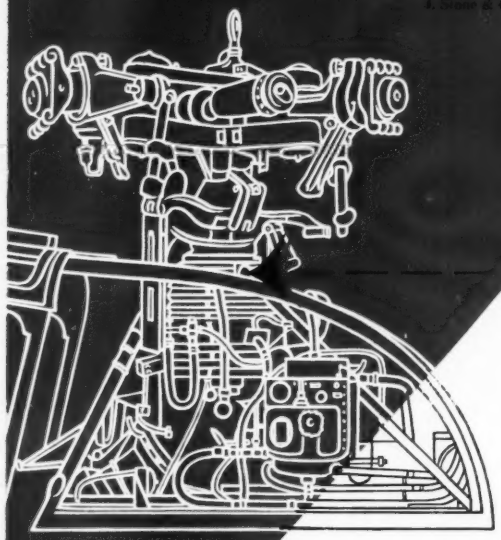

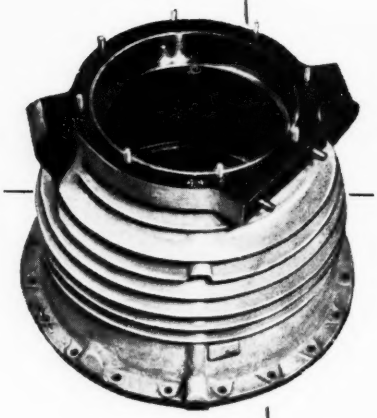

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PUBLISHED MONTHLY AT

4 HAMILTON PLACE, LONDON, W.1

Telephone: Grosvenor 3515-9 *Telegrams*: Didaskalos, Audley, London

SUBSCRIPTIONS: £6 6S. 0D. PER ANNUM, POST FREE. SINGLE COPIES: 10S. 6D., POST FREE.

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All communications for publication in, and advertisements for, the JOURNAL *should be addressed to the Editor,* JOAN BRADBROOKE, *A.R.Ae.S.; all communications on general matters affecting the Society should be addressed to the Secretary,* A. M. BALLANTYNE, *T.D., B.Sc., Ph.D., A.F.I.A.S., A.F.R.Ae.S.*

None of the papers or paragraphs must be taken as expressing the opinion of the Council of the Royal Aeronautical Society unless otherwise stated.

PRINTED BY THE LEWES PRESS (WIGHTMAN & CO. LTD.), LEWES, SUSSEX, ENGLAND, AND PUBLISHED BY THE ROYAL AERONAUTICAL SOCIETY, 4 HAMILTON PLACE, LONDON, W.1, ENGLAND

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***YEAR BOOK—MEMBERS** are reminded that unless their entries for the Year Book are returned by the 31st March 1955, the details given for them in the Year Book may be incorrect and no entry will be made under the Classified Section*

NOTICES

Nominations of Candidates for Council

The following is an extract from the By-Laws:—

"The Twenty-one ordinary members (of the Council) shall be nominated and elected from among the members of the Society. At the date of their election at least ten shall be Fellows, and one at least shall be in each of the following classes: Associate Fellow, Associate and Graduate.

"Of the ordinary members of the Council, that number necessary to create seven vacancies shall retire annually. The retiring members shall be those with the longest service since their last election but they shall be eligible for re-election.

"Nominations of candidates for election to the Council must be received by the Secretary not later than 10th April in each year and shall include statements in writing by the candidates that they are willing to serve. The nomination forms shall be signed by one proposer and two seconders, all of whom shall be Voters."

Nomination forms may be obtained on application to the Secretary.

Eighth Louis Bleriot Lecture

The Eighth Louis Bleriot Lecture will be held on Thursday 17th March 1955 at 6.0 p.m. at the Institution of Mechanical Engineers, 1 Birdcage Walk, S.W.1. Tea will be served at 5.30 p.m. The Lecture, in English, will be by M. Georges Hereil, President, Director-General, Société Nationale de Constructions Aéronautique de Sud Est (S.N.C.A.S.E.), on "Making Aeroplanes Independent of Runways." Members do not require tickets but should apply for tickets for guests who are not members.

The Cantor Lectures

The Three Cantor Lectures of the Royal Society of Arts on "The Mechanical Properties of Metals" will be given on Mondays, 14th, 21st and 28th March 1955, at 6 p.m. Professor Hugh Ford, D.Sc., Ph.D., M.I.Mech.E., will lecture on 14th March, Professor E. N. da C. Andrade, D.Sc., Ph.D., LL.D., F.R.S., on 21st March and Major P. L. Teed, A.R.S.M., F.R.Ae.S., F.I.M., on 28th March. Applications for tickets should be addressed to the Secretary of the Royal Society of Arts, John Adam Street, Adelphi, London, W.C.2.

Institution of Mechanical Engineers

Clayton Lecture

Dr. I. Sikorsky, who has been awarded the 1955 James Watt International Medal by the Institution of Mechanical Engineers, is to give the James Clayton Lecture on the 29th April 1955 at the Institution of Mechanical Engineers, 1 Birdcage Walk, S.W.1, at 5.30 p.m. A limited number of tickets are available for Members of the Society and applications for them should be made to the Secretary of the Society at 4 Hamilton Place, W.1.

Annual General Meeting, 5th May 1955

Notice is hereby given that the Annual General Meeting of the Royal Aeronautical Society, with which is incorporated the Institution of Aeronautical Engineers, will be held on Thursday 5th May at 5.30 p.m. in the offices of the Society, 4 Hamilton Place, London, W.1.

Agenda

1. To read the Notices convening the Meeting.
2. To receive and deliberate upon the Report of the Council on the state of the Society and the Balance Sheets and Income and Expenditure Accounts of the Royal Aeronautical Society and Aeronautical Trusts for the year ended 31st December 1954.
3. To receive the names of those elected to Council for the years 1955-58.
4. To announce the names of Fellows elected by the Council in accordance with By-Law 4.
5. To elect the Auditors for the year 1955.
6. Any other business.

By order of the Council
A. M. Ballantyne,
Secretary.

Note: In accordance with the By-Laws any member whose subscription has not been paid before the first day of April is not entitled to vote.

Light refreshments will be served after the meeting.

Rotterdam Helicopter Congress

A helicopter congress organised by the Rotterdam Helicopter Syndicate is being held in Rotterdam on 27th, 28th and 29th July 1955. The subjects considered will be commercial possibilities of helicopter transport now and in the future and the construction of heliports, together with such factors in the technical development of the helicopter as may influence commercial use of helicopters and the standards for heliports. A number of personalities in the helicopter field from the United States, England, France, Germany, Belgium and the Netherlands have been invited to address the Congress.

Full particulars of the Congress may be obtained from the Secretary of the Rotterdam Helicopter Syndicate, Exchange Building, Coolsingel 58, Rotterdam. As accommodation in Rotterdam is expected to be difficult because the national exhibition E55 is being held at the same time, those interested are advised to apply by 31st March 1955.

Graduates' and Students' Section

A visit has been arranged to Vickers-Armstrongs Ltd. Weybridge Works for Saturday 30th April 1955 at 10.30 a.m. The numbers for this visit are limited and application should be made immediately to the Hon. Visits Secretary, Mr. P. D. Stewart, 217 High Road, East Finchley, London, N.2. Applicants are asked to state their grade of membership of the Society.

Diary

LONDON

10th **March**

MAIN LECTURE AT PRESTON BRANCH.—Structural Safety. Professor A. G. Pugsley, O.B.E. The Bull and Royal Hotel, Preston. 7 p.m.

15th March

SECTION LECTURE.—The Production of Large Forgings. C. Smith and J. Crowther. 4 Hamilton Place, W.1. 7 p.m.

17th March

EIGHTH LOUIS BLERIOT LECTURE, LONDON.—Making Aeroplanes Independent of Runways. Monsieur Georges Hereil, President, Director-General, S.N.C.A.S.E. Institution of Mechanical Engineers, 1 Birdcage Walk, S.W.1. 6 p.m. (Tea 5.30 p.m.)

24th March

MAIN LECTURE.—Analogue Computing Applied to Aeronautical Problems. K. V. Diprose. Institution of Mechanical Engineers, 1 Birdcage Walk, S.W.1. 6 p.m. (Tea 5.30 p.m.)

29th March

GRADUATES' AND STUDENTS' SECTION.—Atomic Energy. Professor J. Rotblat. 4 Hamilton Place, W.1. 7.30 p.m.

31st March

SECTION LECTURE.—Application of Aerial Survey to the Economic Development of a Country. T. D. Weatherhead, O.B.E. 4 Hamilton Place, W.1. 7 p.m.

14th **April**

MAIN LECTURE AT GLASGOW BRANCH.—Mixed Power in Modern Aircraft. M. J. Brennan. The Royal Technical College, Glasgow. 7.30 p.m.

GRADUATES' AND STUDENTS' SECTION.—Radio Aids for Navigation and Blind-landing. Captain F. Ormonroyd. 4 Hamilton Place, W.1. 7.30 p.m.

19th April

SECTION LECTURE.—Thermoelasticity. Professor W. S. Hemp. 4 Hamilton Place, W.1. 7 p.m.

21st April

MAIN LECTURE.—Aeronautical Research in Sweden. Major Bo.K. O. Lundberg. The Institution of Mechanical Engineers, 1 Birdcage Walk, S.W.1. 6 p.m. (Tea 5.30 p.m.)

26th April

SECTION LECTURE.—Aerial Systems for Aircraft. R. A. Burberry. 4 Hamilton Place, W.1. 7 p.m.

30th April

GRADUATES' AND STUDENTS' SECTION.—Visit to Vickers-Armstrongs, Weybridge. 10.30 a.m.

3rd **May**

SECTION LECTURE.—Plastic Structures. G. C. Hulbert. 4 Hamilton Place, W.1. 7 p.m.

4th May

GRADUATES' AND STUDENTS' SECTION.—Technical Aspects of Supersonic Aircraft Design. E. J. Catchpole and W. J. Eggington. 4 Hamilton Place, W.1. 7.30 p.m.

BRANCHES

10th **March**

Preston.—Main Lecture.—Structural Safety. Professor A. G. Pugsley, O.B.E. The Bull and Royal Hotel. 7 p.m.

Yeovil.—Cabin Air Conditioning. Park School, Park Road (off Princes Street), Yeovil. 7.30 p.m.

12th March

Birmingham.—Annual Dinner.

14th March

Halton.—Cabin Pressurisation. F. W. Michael. Branch Hut, R.A.F., Halton. 6.45 p.m.

15th March

Belfast.—Aerial Photography. Charles Brown. Kerr Room, Kensington Hotel, Belfast. 7 p.m.

Manchester.—Helicopters. G. S. Hislop. Reynolds Hall, College of Technology, Manchester. 7.30 p.m.

Coventry.—Meeting. Wine Lodge, Coventry. 7.30 p.m.

16th March

Southampton.—The A.R.A. Wind Tunnel. R. Hills, Institute of Education, University of Southampton. 7 p.m.

Weybridge.—Modern Motor Vehicle Production Methods with Special Reference to Jaguar Cars. J. Silver. Vickers-Armstrongs Ltd., Weybridge. 6 p.m.

17th March

Hatfield.—Combined Meeting with the D.H. Engineering Society. Hatfield Technical College.

18th March

Birmingham.—Trends in Development of Aircraft Electrical and Starting Systems. R. H. Woodall. Birmingham Chamber of Commerce, New Street, Birmingham. 7.30 p.m.

21st March

Halton.—Branch Night. Films and Discussion. Branch Hut, R.A.F., Halton. 6.45 p.m.

Henlow.—Film, "Hell's Angels." Building No. 62, R.A.F. Technical College, Henlow. 7.30 p.m.

Yeovil.—Production Problems on Helicopters. D. C. Collins. Park School, Park Road (off Princes Street), Yeovil. 7.30 p.m.

22nd March

Bristol.—Some of the Problems of Bomber Design. D. Keith-Lucas. Conference Room, Bristol Aeroplane Co., Filton House, Bristol. 6 p.m.

23rd March

Cheltenham.—Some Problems of Bomber Design. D. Keith-Lucas. St. Mary's College, Cheltenham. 7.30 p.m.

Luton.—Annual General Meeting. Staff Canteen. D. Napier & Son Ltd., Luton Airport. 7.30 p.m.

25th March

Southampton.—Annual Dance.

28th March

Halton.—Junior Members' Night. Lecture and Discussion. Branch Hut, R.A.F., Halton. 6.45 p.m.

30th March

Brough.—Annual General Meeting and Film Show. Yorkshire Electricity Board, Ferensway, Hull. 7.30 p.m.

1st **April**

Hatfield.—Annual Dinner.

4th April

Bristol.—Annual General Meeting and Film Show. Conference Room, Bristol Aeroplane Co., Filton House, Bristol. 6 p.m.

5th April

Boscombe Down.—The Light Fighter. W. E. W. Petter, C.B.E. Main Dining Hall, Airmen's Mess, A. and A.E.E., Boscombe Down. 5.30 p.m.

Glasgow.—The Twin Pioneer: Design, Production and Test Flying. T. B. Lyon, J. Hislop and N. J. Capper. Royal Technical College, Glasgow. 7.30 p.m.

6th April

Chester.—Operation of Jet Engined Aircraft. The Grosvenor Hotel, Chester. 7.30 p.m.

Hatfield.—Salvage Operation of the Imperial Airways Flying Boat "Corsair" in the Belgian Congo. R. Sisson. de Havilland Restaurant, Hatfield. 6.15 p.m.

Luton.—Aerial Photography. Russell Adams. Staff Canteen, D. Napier & Son Ltd., Luton Airport. 7.30 p.m.

Weybridge.—Some Maintenance Aspects of Viscount Operation. E. R. Major. Vickers-Armstrongs Ltd., Weybridge. 6 p.m.

11th April

Halton.—Films: "Measuring Craftsmanship," "British Adventure," "Cinemagazine No. 13." Branch Hut, R.A.F., Halton. 6.45 p.m.

13th April

Glasgow.—Annual General Meeting. St. Enoch Hotel, Glasgow. 7.30 p.m.

Preston.—Annual General Meeting and Film Show. R.A.F. Association, Preston. 7.30 p.m.

14th April

Glasgow.—Main Lecture.—Mixed Power in Modern Aircraft. M. J. Brennan. The Royal Technical College, Glasgow. 7.30 p.m.

19th April

Belfast.—Annual General Meeting. Kerr Room, Kensington Hotel, Belfast. 7 p.m.

20th April

Coventry.—Annual General Meeting and Films. Wine Lodge, Coventry. 7.30 p.m.

Hatfield.—Discussion Evening. de Havilland Restaurant, Hatfield. 6.15 p.m.

21st April

Southampton.—Nuclear Power Plants. Institute of Education, University of Southampton. 7 p.m.

Yeovil.—Carrier Operations (Wyvern). Lt.-Cdr. C. E. Price. Park School, Park Road (off Princes Street), Yeovil. 7.30 p.m.

27th April

Gloucester.—Annual General Meeting and Film Show. The Wheatstone Hall, Brunswick Road, Gloucester. 7.30 p.m.

28th April

Yeovil.—Annual General Meeting. Park School, Park Road (off Princes Street), Yeovil. 7.30 p.m.

4th **May**

Luton.—Naval Aircraft. Staff Canteen, D. Napier & Son Ltd., Luton Airport. 7.30 p.m.

Weybridge.—Annual General Meeting, Vickers-Armstrongs Limited, Weybridge. 6 p.m.

9th May

Halton.—The History of Parachuting and Ejection Seats. A./A. R. D. Trender (Junior Member). Branch Hut, R.A.F., Halton. 6.45 p.m.

News of Members

John Bright (*Student*) has resigned his post with British Messier on being appointed a Senior Scientific Officer in the Royal Naval Scientific Service.

E. C. Carter (*Associate Fellow*) has been appointed Senior Aerodynamicist to the Aircraft Research Association, Bedford.

A. V. Cleaver (*Fellow*) has been appointed Chief Engineer of the Rocket Division of the de Havilland Enterprise.

Handel Davies (*Fellow*) has been appointed Scientific Adviser to the Air Ministry. He was formerly Chief Superintendent of the Aeroplane and Armament Experimental Establishment at Boscombe Down.

Vice-Admiral Caspar John (*Associate Fellow*) is to be Flag Officer Air (Home) from 1st June 1955. His present appointment is that of Deputy Controller of Aircraft Production at the Ministry of Supply.

N. Green (*Associate Fellow*) has been appointed Assistant Chief Designer to Aviation and Engineering Projects Ltd.

G. W. Hall (*Associate Fellow*) has been appointed Assistant Managing Director of the Fairey Aviation Company.

J. Kirby (*Associate Fellow*) has been appointed Senior Project Stress Engineer at Jarry Hydraulics Ltd., Montreal.

A. E. Maine (*Associate*) has been appointed Project Engineer with the de Havilland Aircraft Company of Canada.

W. N. Neat (*Associate Fellow*) has been appointed Deputy Chief Engineer of the Rocket Division of the de Havilland Enterprise.

Wing Cmdr. V. C. Otter (*Associate Fellow*) has relinquished Command of the Central Servicing Development Establishment and has been appointed Senior Technical Officer at the Royal Air Force station at Wildenrath.

W. H. Parr (*Associate*) has been appointed Local Director and Works Manager of the Sanbra Engineering Company, Birmingham.

C. G. Pullin (*Fellow*) has been appointed Director and Technical Consultant to Winger Limited of Rochester.

D. N. Walker (*Associate Fellow*) has relinquished his position with the National Coal Board and has been appointed Chief Engineer to the Wilmot Breeden Group of Companies, which includes Telehoist Limited of Cheltenham.

W. J. Woods (*Associate Fellow*) has been appointed Chief Stressman of Aviation and Engineering Projects Ltd.

Gerald R. Wooll (*Associate*) has been appointed Managing Director and a Member of the Board of The Kaman Aircraft Corporation, Connecticut.

Acknowledgment

The Council acknowledge with grateful thanks the return of back numbers of the Journal and the gift of sundry papers from Mr. W. T. Reid, *Associate*, is acknowledged with thanks.

Elections

The following is a list of new members and transfers of membership of the Society:—

Associate Fellows

David Geoffrey Ainley (*from Student*)
Reginald Arthur Bridgeman (*from Graduate*)
John Farrall Bush
Frederick James Canham (*from Associate*)
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Richard Henry Collins (*from Graduate*)
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The Journal of the Royal Aeronautical Society

WITH WHICH IS INCORPORATED THE INSTITUTION OF AERONAUTICAL ENGINEERS

VOLUME 59 MARCH 1955 NUMBER 531

Boundary Layer Control

by

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(Director of Research, Handley Page Ltd.)

The 924th Lecture to be given before the Royal Aeronautical Society was held at the Institution of Mechanical Engineers, Birdcage Walk, London, S.W.1, on 11th November 1954, and was presided over by Sir Sydney Camm, C.B.E., F.R.Ae.S., President of the Society. Introducing the Lecturer, Dr. G. V. Lachmann, F.R.Ae.S., the President said that his work with Handley Page Ltd. for so many years was well known and his subject—"Boundary Layer Control"—was one that had been discussed and considered for many years. Now it seemed that they must do something about it; without question, future designs would demand some form of boundary layer control, if only from the lift augmentation point of view. As Dr. Lachmann said, it was the gas turbine engine which made it possible now.

Introduction

It has almost become a tradition in recent years to begin a paper on boundary layer by paying a tribute to the great Prandtl and his famous paper[1] in which he introduced the conception of boundary layer into Fluid Dynamics.

The year 1954 is, however, not only memorable to mark the passing of fifty years since Prandtl's classical demonstration of the effect of boundary layer suction on the flow pattern around a cylinder; it is also memorable because in 1954 most convincing demonstrations of practical applications of boundary layer control for aircraft have taken place. I am referring, in particular, to the demonstrations with the Attinello flap in the United States which mark the introduction of one form of boundary layer control as an engineering and practical reality.

In coming years the present period in aviation may well be recognised as one where an important evolution of design philosophy took place.

In recent years a new school of thought has been growing up leading away from the classical conception of the aeroplane where the power plant serves solely for the production of thrust to pull the lift producing wings through the air.

The new conception aims at a closer integration of power plant and wings and at harnessing the power plant to the combined task of lifting and propulsion.

This conception has been stimulated by the introduction of the gas turbine and, broadly speaking, the evolution is one leading away from the motorised glider as pioneered by the Wright Brothers to what might be termed a "winged turbine."

The new philosophy has many branches and is not confined to fixed-wing aircraft. The extreme manifestation of this philosophy is the production of lift directly without the intermediary of circulation (flying bedstead).

Another branch of this new outlook consists in by-passing a small proportion of the available power for controlling the boundary layer and the circulation around the wings.

Modern methods of boundary layer control for the purpose of preventing separation are based on the principles of either infusing kinetic energy into the flow, or by removing boundary layer through suction.

The possibility of laminarising a boundary layer through suction was first envisaged by Griffith and Meredith[2] in 1936, but the first experiments were not successful. The same theoretical conclusions were reached, a few years later, by Holstein[3] in Germany, who succeeded in demonstrating the possibility of laminarising the boundary layer through suction.

While research workers in many countries became fascinated with the idea of developing methods of boundary layer control, both for the purpose of lift increase and laminarisation, the majority of aircraft designers were inclined to consider any suggestions of boundary layer control as purely academic and impracticable.

With the introduction of the gas turbine for aircraft propulsion, the practical outlook has changed decisively since now a useful reservoir of compressed air exists for directing a jet of air into the boundary layer, or for energising suitable suction devices. Besides, the introduction of swept-back wings, thin aerofoils and increasing wing loadings have emphasised the need for higher lift coefficients than were hitherto available with conventional methods.

Notation

A aspect ratio
a speed of sound
a_1 lift curve slope
C_{D_0} parasitic drag coefficient
C_L lift coefficient
C_{Pd} duct pressure coefficient
C_Q suction coefficient
C_μ blowing momentum coefficient
c wing chord
c_1 effective extended wing chord, *e.g.* with Fowler flap
c_f flap chord
D total drag
D_B body drag
D_i induced drag
D_0 parasite drag
D_p equivalent pump drag
D_W wake drag of wings and tail (no suction)
H altitude
k induced drag factor
L lift
l distance between suction strips (*see* Fig. 23)
M Mach number
m mass flow
p_d duct pressure
p_0 undisturbed stream or atmospheric ambient pressure
Q sucked volmetric flow
q_0 undisturbed stream dynamic pressure
q_{0opt} undisturbed stream dynamic pressure when total drag is minimum
R Reynolds number
R_c Reynolds number based on chord and undisturbed stream velocity $= \dfrac{U_0 c}{\nu}$
R_l Reynolds number based on strip spacing and local velocity $= \dfrac{U.l}{\nu}$
R_δ^* Reynolds number based on boundary layer displacement thickness and local velocity $= \dfrac{U_\delta^*}{\nu}$
$(R_\delta^*)_{crit}$ critical value of $R_\delta^* = \left(\dfrac{U_\delta^*}{\nu}\right)_{crit}$
$(R_\theta)_{crit}$ critical value of Reynolds number based on boundary layer momentum thickness and local velocity $= \left(\dfrac{U\theta}{\nu}\right)_{crit}$
S sucked surface or wing area
s width of suction strip (*see* Fig. 23)
T engine thrust
t maximum thickness of an aerofoil section
U local stream velocity
U_0 undisturbed stream or flight velocity
U_β velocity component of boundary layer in direction perpendicular to outer streamline (*see* Fig. 33)
v suction inflow velocity
v_j velocity of jet
W weight at take-off
w rate of climb
x a distance measured in the direction of flow
δ boundary layer 99 per cent. thickness
δ^* boundary layer displacement thickness
δ_1^* boundary layer displacement thickness just after a suction strip
δ_0^* boundary layer displacement thickness at transition on a smooth surface
δ_T^* boundary layer displacement thickness at transition on a rough surface
δ_{Tl}^* lower critical limit of boundary layer displacement thickness
δ_{Tu}^* upper critical limit of boundary layer displacement thickness
δ_u^* boundary layer displacement thickness just before a suction strip
δ_x^* boundary layer displacement thickness at position x
δ_ε^* boundary layer displacement thickness at position of roughness element
ΔC_L lift coefficient increment due to flap deflection
ΔC_{LT} theoretical lift coefficient increment due to flap deflection according to Glauert's thin aerofoil theory
ΔD_W wake drag of wing and tail surfaces (laminarised aircraft)
Δp a pressure difference
ε height of a roughness element
η flap angle
η_F Froude efficiency
η_0 overall efficiency of a propulsion system
η_p pump efficiency
θ boundary layer momentum thickness
θ_1 boundary layer momentum thickness just after a suction strip
θ_u boundary layer momentum thickness just before a suction strip
λ_1 a function of c_f/c occurring in Glauert's thin aerofoil theory
ν kinematic viscosity
ρ_0 undisturbed stream or atmospheric ambient density
χ a Reynolds number $= \dfrac{U_{\beta max}.\delta}{\nu}$
ω_t slot width
$\dfrac{dp}{dx}$ a pressure gradient on the surface

PART I

1. Boundary Layer and Circulation Control through Blowing and Sucking

1.1. GENERAL PRINCIPLES AND DEFINITIONS

It is proposed to deal first with the application of boundary layer control to prevent separation of flow from the surface of an aerofoil, for example at the sharp nose of an aerofoil of small thickness/chord ratio.

By preventing separation and premature stall the lift increase will continue to larger angles of attack than for the ordinary section.

The slotted wing exemplifies the first practical form of an automatic boundary layer control.

Boundary layer which is formed over a slat at the leading edge flows off at its trailing edge when the slot opens, and the secondary flow through the slot prevents this separated boundary layer from interfering with the flow over the main wing (Fig. 1).

The circulation formed around the slot (in the open position) reduces the pressure gradient over the nose of

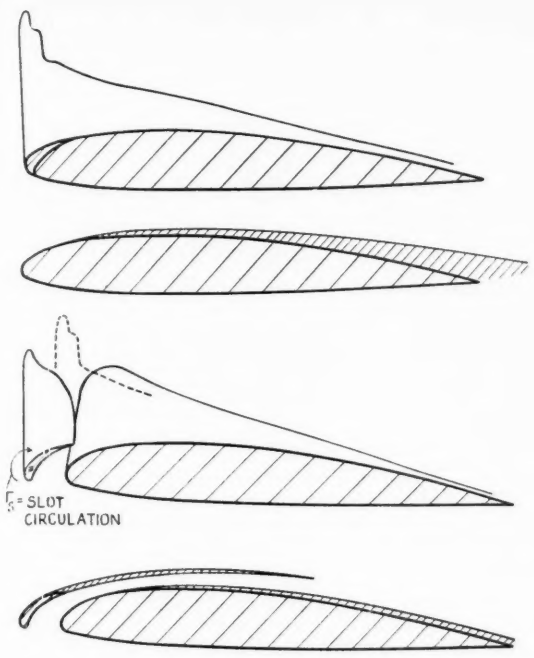

FIGURE 1. Principle of slot.

the main wing and makes it, therefore, easier for the boundary layer to flow over it without separating from the surface. Equally well, separation is delayed on the slat itself, since here too the pressure gradient from slat leading edge to slat trailing edge is reduced, the latter being placed in the low pressure region which occurs over the nose of the main wing.

The slotted flap improves the ordinary flap in the same manner, by delaying separation at the flap hinge and by allowing a higher circulation to develop over the part of the wing ahead of the flap, which fulfils the role of the slat.

The stream of air coming from a slot has the same effect as a jet pump and infuses kinetic energy into the boundary layer. The tendency of the flow to separate from the wing surface depends on whether the retarded boundary layer air is entrained in sufficient quantities into the main stream.

This conception of the slot effect led Professor A. Baumann to the method of blowing out a stream of air at suitable points on the upper side of the wing to accelerate the boundary layer. While in the case of a slotted wing the velocity of the secondary flow through the slot is limited by the pressure difference between upper and lower surface, with blow-out the velocity of the secondary stream of air can be chosen at will. Otherwise, there is no fundamental difference between the two methods.

Separation can thus be prevented either by established methods of slots or nose flaps, or alternatively by removing excessive boundary layer by suction through porous material or through a suction slot near the leading edge, or alternatively, by infusing kinetic energy into the boundary layer and improving the process of entrainment by blowing air out tangentially to the surface contour.

By preventing separation at the leading edge and premature stalling of the wing the linearity between lift and angle of incidence can be continued to larger angles of incidence (Fig. 2).

The classical method of circulation control, in contrast to extending the usable range of the lift curve, is the trailing edge flap. When an ordinary type of trailing edge flap (slotted or unslotted) is deflected at constant angle of incidence, circulation will increase and the lift curve will shift to the left in Fig. 2.

However, the observed increment ΔC_L due to large flap angles will be substantially less than one would expect from the application of Glauert's theory for hinged flaps. The reason is the peculiar pressure distribution with strong adverse pressure gradients on the upper surface of the aerofoil leading to separation of the boundary layer (Fig. 3). The actual pressure distribution which will normally develop is shown in dotted lines.

Separation at the flap can be prevented either by sucking away boundary layer near the flap hinge, or by

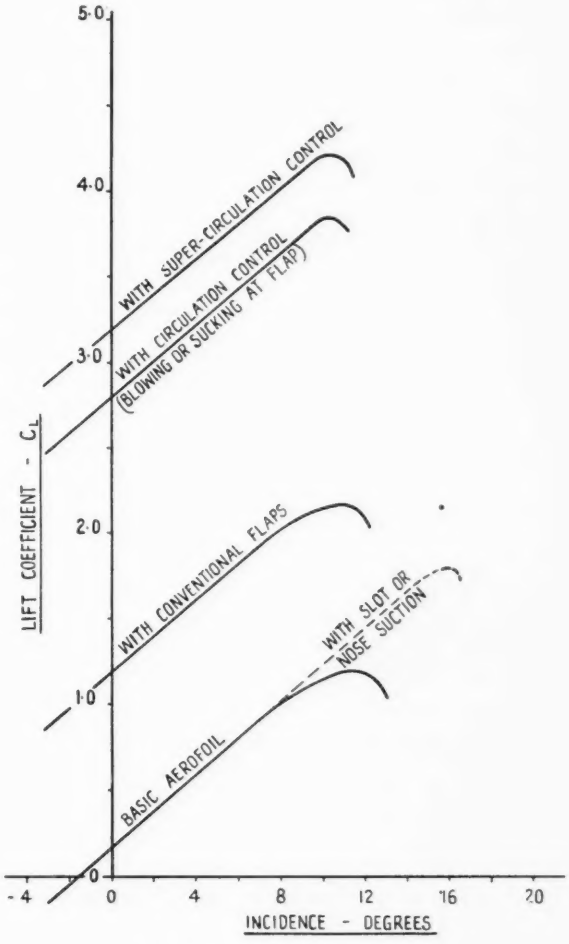

FIGURE 2. Generalised lift curves.

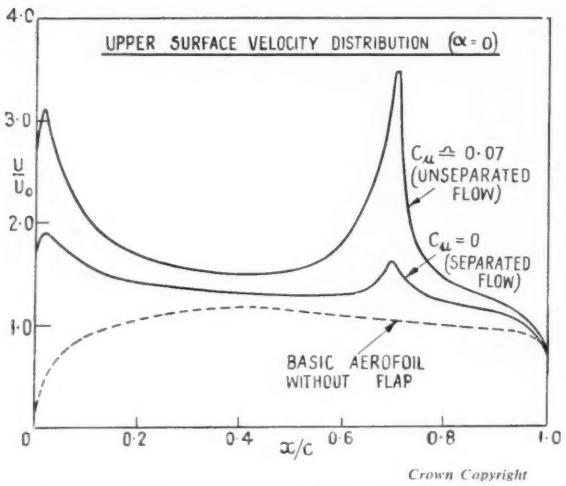

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FIGURE 3. Pressure distribution over upper surface of a flapped aerofoil.

blowing a jet of air through a slit ahead of the flap. The Coanda effect makes this secondary flow adhere to the curvature of the flap nose and entrains the retarded air in the region of separation.

Thus it is possible to bend the flow over the flap and to achieve the lift increase ΔC_{LT}, which theory predicts for a flap of equal flap/chord ratio and flap angle.

Figure 4 shows the various methods of boundary layer control applied to an aircraft wing, and Fig. 2 shows the generalised effect of these methods on lift and angle of incidence.

When boundary layer control through suction is applied it has become customary to express the intensity of suction by a coefficient:

$$C_Q = Q/(U_0 S)$$

On the other hand we have

$$C_P = \Delta p / q_0$$

where

$$\Delta p = p_a - p_0$$

$$q_0 = \tfrac{1}{2}\rho_0 U_0^2$$

The power for pumping follows from

$$\text{H.P.} = \frac{\Delta p}{550}\frac{Q}{\eta_p} = \frac{C_{Pa} C_Q}{\eta_p}\frac{\rho U^3 S}{1{,}100}$$

For blowing systems a blowing momentum coefficient

$$C_\mu = \frac{m v_j}{S q_0}$$

introduced by Poisson-Quinton[4] has been generally adopted.

A method of increasing circulation without using a flap originated in Germany during the 1939-45 War (Regenscheit[5]). The method consists in sucking boundary layer and part of the potential flow into a relatively large slot situated near the trailing edge of the unflapped aerofoil.

Dr. John Williams[6] of the Aerodynamics Division of the National Physical Laboratory has recently published a very valuable analysis of available aerodynamic data from various sources on blowing over trailing edge flaps for increasing lift.

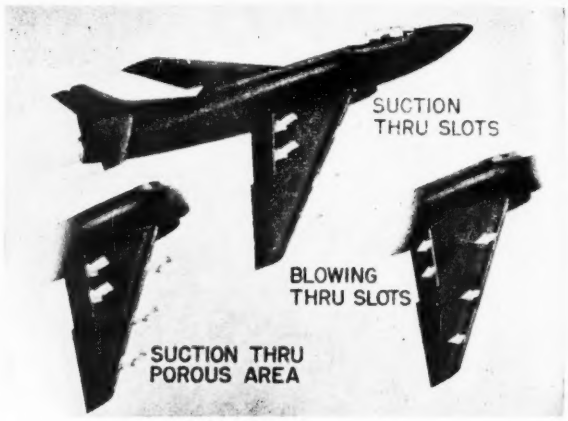

FIGURE 4. Various methods of boundary layer control applied to an aircraft wing.

Lift, drag and pitching moment data are correlated in terms of the blowing momentum coefficient C_μ which is a far more satisfactory unique parameter than the

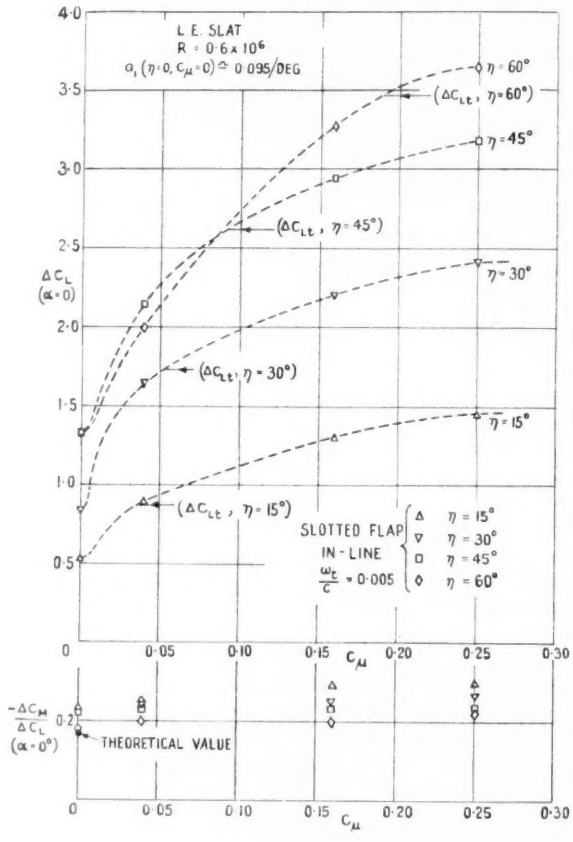

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FIGURE 5. Analysis of German two-dimensional tests on NACA 0009 aerofoil with 25 per cent. slotted T.E. flaps.

blowing quantity coefficient $C_Q = m/(\rho_0 U_0 S)$ and the blowing pressure coefficient $C_{Pd} = (p_d - p_0)/q_0$.

As a unique datum for comparing the sectional lift increments obtained with the various flap configurations, the lift increments ΔC_{LT} given by Glauert's thin aerofoil theory for the appropriate flap angle η and flap/chord ratio c_f/c have been introduced.

Interpreting the effect of flap deflection as a change in the no-lift angle of the section the maximum theoretical value ΔC_{LT} (without separation) is defined as

$$\Delta C_{LT} = a_1 \lambda_1 (c_f/c)\, \eta$$

The function $\lambda_1 (c_f/c)$ has been given values derived by Glauert's mean-line theory. Typical values are 0·55, 0·61 and 0·66 for c_f/c ratios of 0·20, 0·25 and 0·30, respectively.

If there is substantial rearward movement of the flap as it deflects, then the ratio c_f/c_1 should be taken instead of c_f/c, where c_1 is the effective (extended) wing chord.

Figure 5 is a typical example of an analysis on this basis applied to German two-dimensional tests on N.A.C.A.0009 aerofoil with 25 per cent. slotted T.E. flaps with blow-out. These experiments took place during the 1939-45 War. In this figure the values for ΔC_{LT} are marked. It should be noted that if C_μ is increased beyond the value corresponding to ΔC_{LT} where the flow is completely attached over the flap, a further lift increase occurs, but at a reduced rate.

This further improvement is interpreted by American research workers as a "super-circulation." It may be regarded as arising from an effective jet extension of the flap chord and from the vertical component of the jet reaction. In addition, at large wing incidences, the flow separation on the wing ahead of the flap will be reduced by induction effects from the high velocity jet.

The values of C_μ needed in practice on two-dimensional and finite wings to provide a "datum" lift increment corresponding nominally to unseparated flow over the flap, offer a good criterion by which to judge the economy of various methods of blowing over a flap of given c_f/c ratio at a given value η.

In Dr. Williams'[6] opinion there is need for further high-lift tests with blowing over T.E. flaps, particularly on the straight and swept-back wings, and at high Reynolds numbers.

In my opinion, there is also room for the study of other methods preventing separation from the flap.

2. Brief Review of Developments of Boundary Layer and Circulation Control for High Lift in Various Countries

2.1. GREAT BRITAIN

An aircraft with boundary layer control through perforations in the skin was built and flown by Miles[7] in 1938. This attempt was probably a little too far

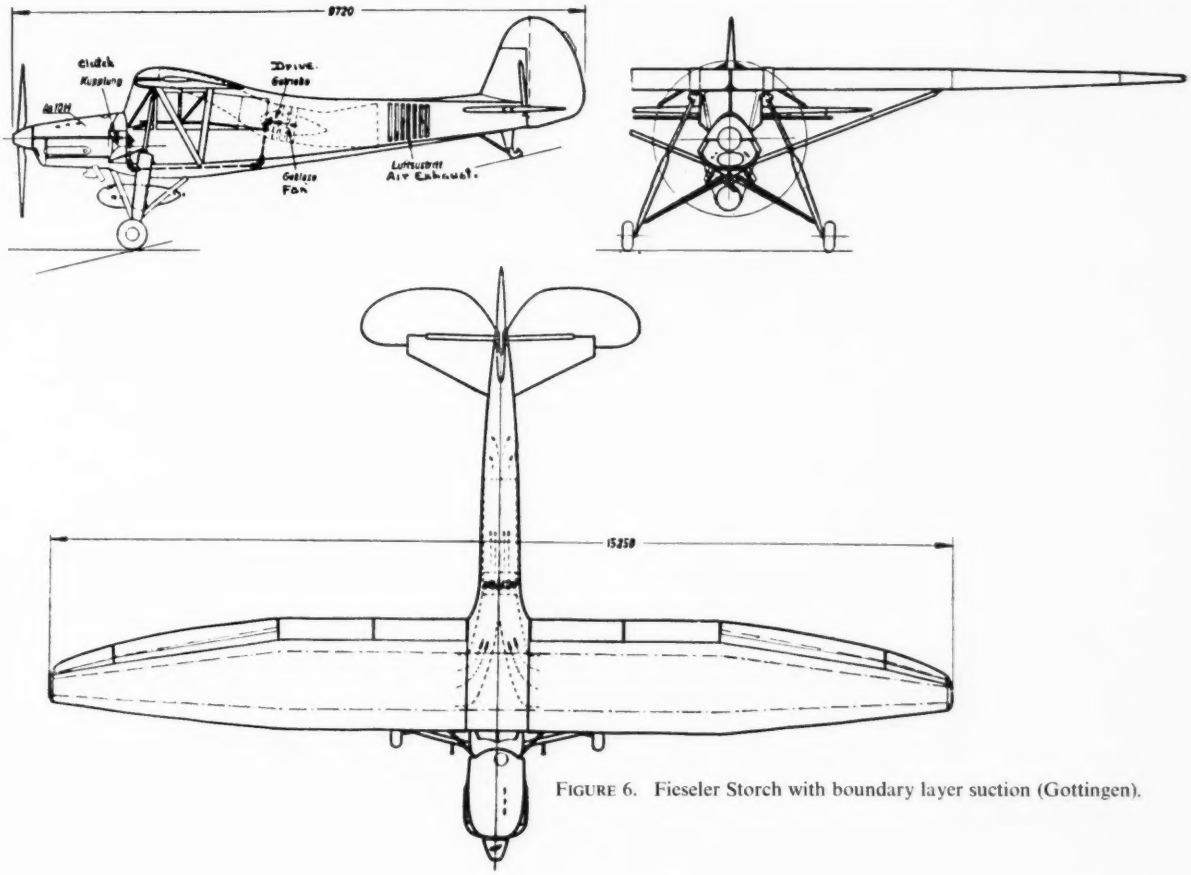


FIGURE 6. Fieseler Storch with boundary layer suction (Gottingen).

FIGURE 7. Details of suction ducts on Fieseler Storch.

ahead of its time, and I do not know whether it was designed for drag reduction or lift increase, or both.

Wind tunnel experiments took place in 1939 and in 1941 at R.A.E. (Lyon, Hills, Barnes and Adamson)[8]. The aerofoil had a thickness/chord ratio of 18 per cent. (N.A.C.A.2218) and various types of plain flaps were tried in conjunction with a blow-out slot of 0·00830 in. width.

Lift increments ΔC_L were measured for flap angles of 60° and 53° at three different angles of incidence and up to very high values of C_L (0·5).

No further research work seems to have been done until 1950 when the method of blowing out over a flap found renewed interest for potential application to swept-back wings. I refer to wind tunnel tests by W. H. Paine, of Westland Aircraft Ltd.

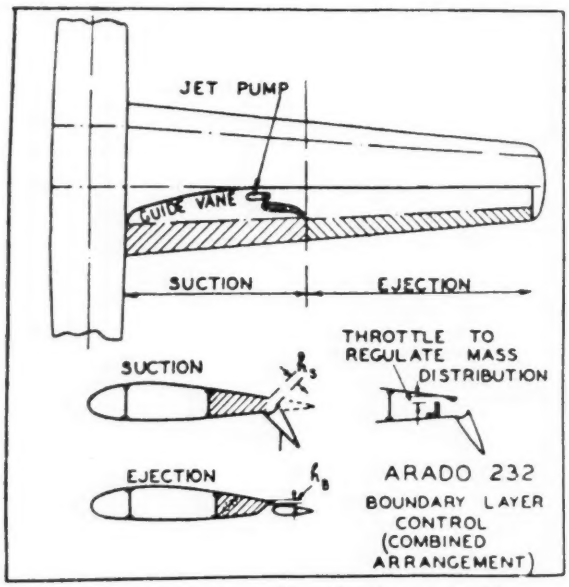


FIGURE 8. Arado system of boundary layer control by sucking and blowing.

2.2. GERMANY

O. Schrenk[9] undertook a great deal of wind tunnel research on boundary layer suction through slots at Göttingen between 1930 and 1936. Göttingen has always held a traditional position in the forefront of boundary layer research, and this research work by Schrenk, aiming at a practical application of Prandtl's classical conception of boundary layer suction, was greatly stimulated by Professor Betz[10] and Dr. Ackeret[11] (now Professor at Zurich E.T.H.).

The effect of blowing through the trailing edge of the main surface over the flap (Junkers-type slotted flap) was studied at the Technical University of Hanover in 1938[12].

During the 1939-45 War a great deal of research with circulation control through blowing and suction was done by Schwier[13], Regenscheit[14], Ehlers[15] and Walz[16].

First flight tests with boundary layer suction were undertaken in 1936 on a Fieseler Storch, modified for this purpose. In 1941 these tests were continued with an improved version (Figs. 6 and 7). Application of boundary layer control was planned for the Dornier 24 flying boat, the Messerschmitt Me.109 fighter and the Arado 232 troop transport and 234 turbo-jet bomber.

Both the Arado and Dornier aircraft were to use a combination of sucking and blowing; the air over the inner half of the span was sucked away and the same air was then expelled through a slot over the outer half of the span. Thus the air was utilised twice. This resulted in a corresponding improved economy of ducting and pump power. Although this system is generally connected with the name Arado, it is worth mentioning that it is based on a patent by Herbert Wagner and von Doepp (Fig. 8).

On the Me.109, boundary layer control was to be effected solely by blowing out over the flaps. The need for increasing the lift coefficient resulted from the use of laminar flow wing sections of the symmetrical type having a thickness chord ratio of only 9 per cent., with

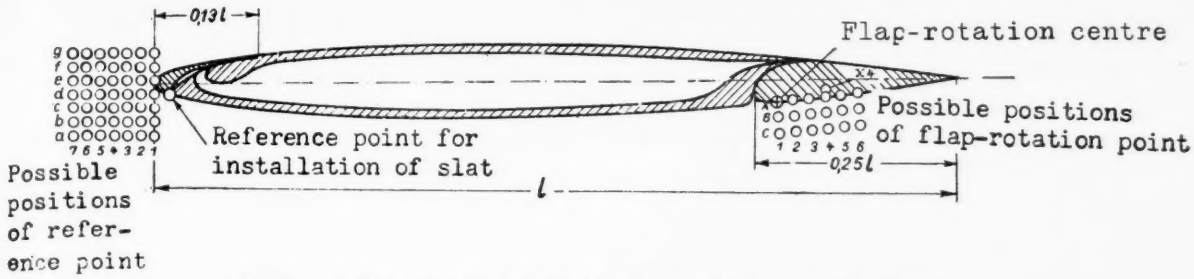


FIGURE 9. Method of blowing air out of a wing fitted with slots and slotted flaps.

the maximum thickness at 40 per cent. of the chord. Originally it was intended to provide the wing with a slotted flap and a slat, and the influence due to the standard means was to be intensified by blowing out at the slat as well as at the flap. By blowing air into the slot formed between the slat and the main wing a considerable increase of lift could be obtained (Fig. 9). However, it was found that it was more economical to produce a high value of ΔC_L when only the flap was subjected to this process, than when the artificial blowing was used simultaneously on flap and slat. Air was to be supplied by two blowers located inside the wings and driven by extension shafts from the main power plant, a reciprocating engine.

When investigating the effect of blow-out on separation over the flap surface, it was observed that although the inner layer of flow did not seem to separate from the surface of a conventionally designed slotted flap, the lift increase did not have the expected magnitude. A careful study of the flow conditions showed that in such cases only a very thin layer of the flow adhered to the upper surface of the flap. The entrainment of the boundary layer was not strong enough to influence the direction of the main stream sufficiently to cause the desired increase of the lift coefficient (Fig. 10).

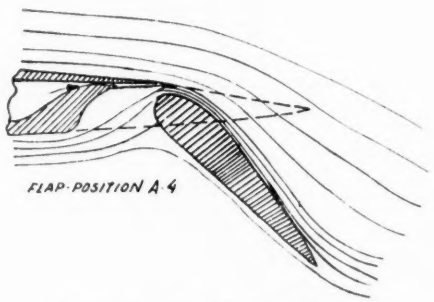


FIGURE 10. Slotted flap with blow-out.

A series of flap positions was tried where the upper surface of the flap at larger angles of deflection protruded beyond the contour of the profile and did not recede below the contour, as in a normal slotted flap (Fig. 11). It was found that now it was possible to prevent the main air stream from separating, even at flap deflections of the order of 60 degrees. A closer study of the air flow indicated in this case that the whole outer air flow remained parallel to the upper contour of the flap, and that the effect of blowing was not confined to a thin layer of air in direct contact with the surface only.

It was concluded that with the flap contour protruding beyond the wing contour, the ejected air from the inside of the wing had a better chance to mix with the boundary layer flowing over the upper surface of the flap and to ensure a better entrainment with the main flow.

The end of the Second World War put an end to these German attempts for an engineering solution of boundary layer and circulation control.

Although the results of this German work of blowing and sucking became available to the Allies immediately after the war, it found relatively little response in Great Britain. This may have been due to the poor translation of the original German reports and bad representation of the results.

A revival of interest was caused by Sir Frederick Handley Page's "Louis Bleriot" lecture in 1950[17], in which he drew attention to the potentialities of boundary layer control for lift increase and the resulting reduction of take-off distances and runway lengths for modern air liners.

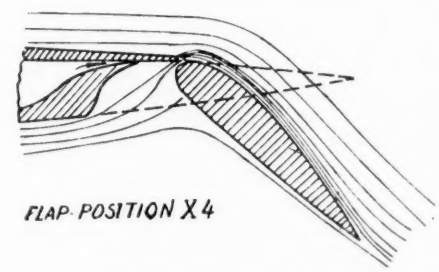
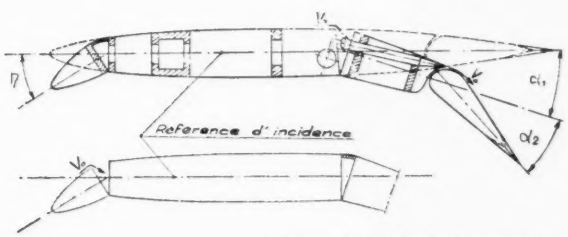


FIGURE 11. Slotted flap with blow-out with flap contour protruding beyond the contour of the profile.



Courtesy of La Recherche Aeronautique

FIGURE 12. Swept wing with combined blow-out and suction (Poisson-Quinton).

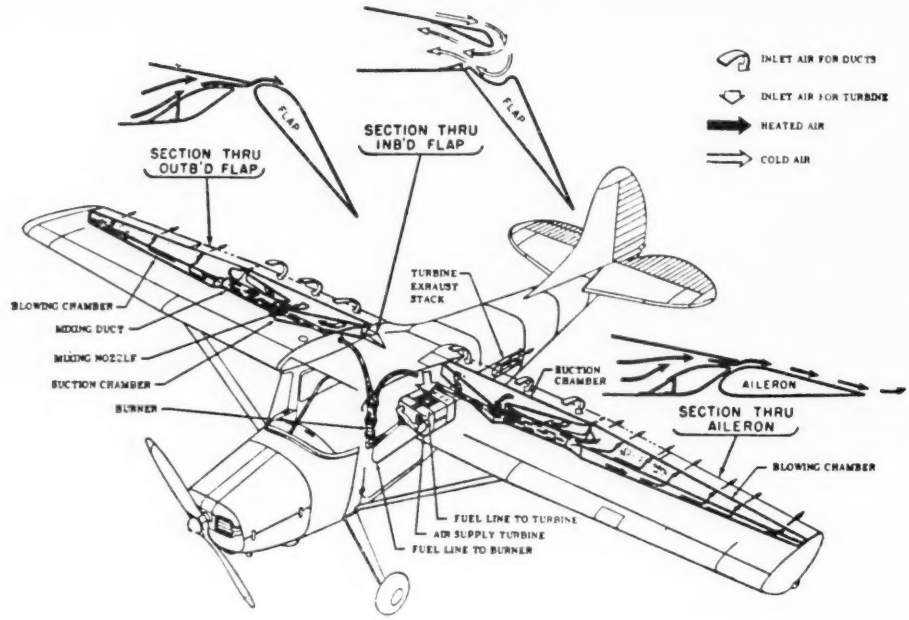


FIGURE 13. Cessna 170 aircraft with boundary layer suction.

2.3. FRANCE

The German research work was followed up after the 1939-45 War in France by O.N.E.R.A. Poisson-Quinton applied the principle of combined suction and blowing to a thin swept fighter wing, introducing the very productive idea of bleeding air from the jet engines to energise the driving nozzles of a series of flattened jet pumps which, in turn, exhausted over the flap (Fig. 12). Double flaps were arranged to permit suction at the first opening of the double flap from the suction side of the pump, with subsequent exhaustion of the air over the rear flap. Suction was also applied at the drooped nose to prevent separation of flow at the nose of the thin swept wing. With this arrangement a lift increase $\Delta C_L = 1{\cdot}36$ was obtained.

2.4. UNITED STATES

The Arado system of sucking and blowing was demonstrated on the American Cessna 170 aircraft (Fig. 13). Aerodynamically speaking, this arrangement was a close copy of the original German scheme; but in place of a jet pump energised by hydrogen-peroxide, a unit supplying hot, highly pressured gases was used. This device was the result of developments by Razak under a Navy contract. The modified Cessna, designated Cessna 309, was demonstrated at the Naval Air Station at Anacostia and showed complete controllability at a speed 11 m.p.h. below the stalling speed of the commercial version at the same weight. The take-off distance was about 40 per cent. shorter than for the normal Cessna 170.

The Cessna aircraft was also used for tests sponsored by N.A.C.A. where the nose of the wings was covered with porous Monel cloth and where suction was applied in order to prevent separation at the nose. Since the aerofoil of the Cessna wing is of a type where one might expect separation to start at the trailing edge, it is not readily understood what the aerodynamic purpose of these experiments was.

The basic methods of circulation control developed by Ehlers, Regenscheit and Schwier formed part of the Low-Speed Flight Research Programme of the Office of Naval Research.

Figures 16 and 17 show the ratio of local to free-stream dynamic pressure without blowing and with blowing, and with and without suction at the flap. Without blowing, the ratio of dynamic pressure is shown by the circle symbol, while the increase over the upper surface and decrease on the lower surface is shown by the triangular symbol. It should be noted that with this type of slot suction a very large slot and very large suction quantities were found necessary.

It would seem possible to achieve a more economical method of suction to prevent flow separation at the flap in a different manner.

The most successful demonstration of a very practical engineering solution of a high lift system using the blow-out method only, without any suction, was given in 1954 in the U.S.A. on a Grumman "Panther" (Figs. 14 and 15). Mr. John Attinello, of the Bureau of Aeronautics, was responsible for this work. Although exact details cannot be disclosed yet, one can infer from published Press reports that the installation was of extreme simplicity.

Not only was a very substantial reduction of take-off speed obtained (20 knots), but the aircraft was able to take off with a bomb load increased by 3,000 lb. Pilots were highly satisfied with the aircraft and mentioned that controllability at low speed was greatly improved and gave them great confidence to fly the aircraft right down to the lowest possible speeds. The sweeping success of this demonstration had a far reaching effect on the rest of the U.S. Aircraft Industry.

The particular interest of naval circles in a simple

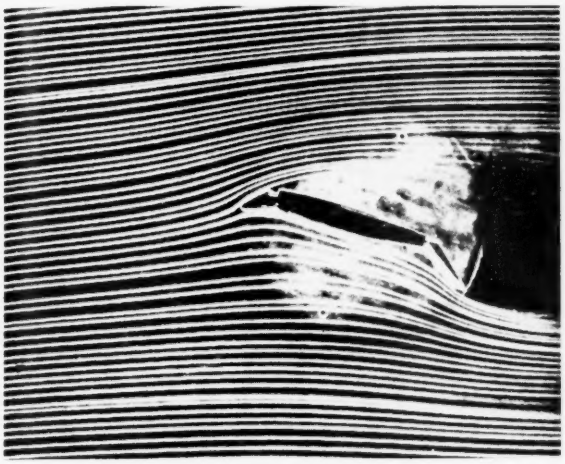


FIGURE 14. Flow field about an aerofoil with ordinary flap.

FIGURE 15. Flow field about an aerofoil fitted with Attinello flap (as installed in a Grumman Panther).

high-lift system which allows reduction of take-off and landing distances is obvious. By reducing take-off speed the same aircraft can be launched from a smaller catapult, or with less wind over the carrier deck; or an aeroplane can be loaded up to a higher gross weight and still be launched by the larger catapult.

In a paper on boundary layer control and super circulation presented by John Attinello at the Washington, D.C., Section of the Institute of the Aeronautical Sciences, on the 3rd March 1953, the author gave a comparison between a "Jato" system (rocket-assisted take off) and the Arado system on a fuel consumption basis. According to this comparison, and at a rate of 5 lb./sec., "Jato" reduced the 660 ft. ground run by only 90 ft., whereas the Arado system cut the distance by 360 ft.

Curves indicating take-off distance over a 50 ft. obstacle show the same trend at low rates. "Jato" becomes equal to the high-lift system at very high consumption rates of the order of 12 lb./sec. There is, though, no use for "Jato" when landing, while the boundary layer system can be used for reducing landing speed.

Although the Attinello flap may not be the only form of application in the future—it is not the most economical one from the point of view of mass-flow requirement—it certainly is one which may be applied very readily to all sorts of modern aircraft.

A brief survey of possible applications of circulation control to existing and future aircraft and of the considerations engendered may be indicated.

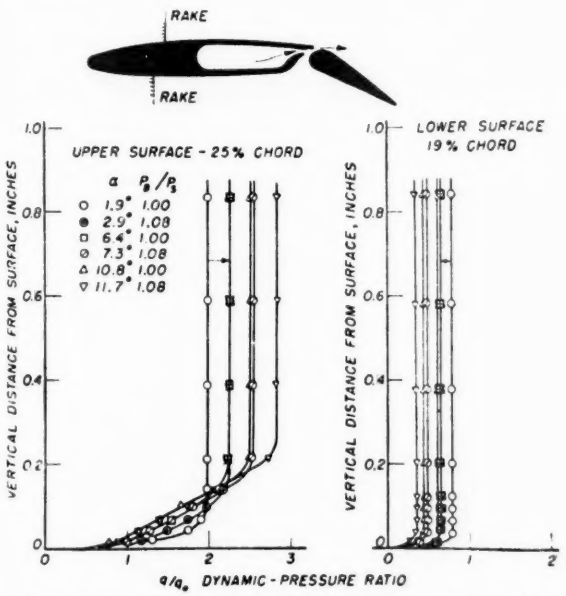


FIGURE 16. Blow-out ratio of local to free stream dynamic pressure without and with blowing.

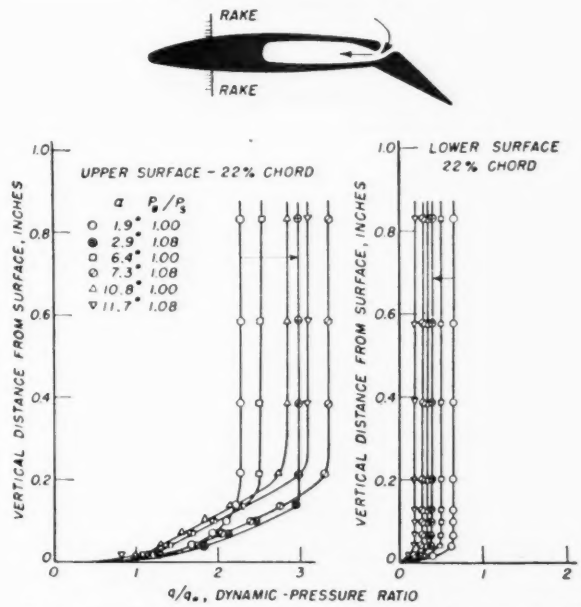


FIGURE 17. Ratio of local to free stream dynamic pressure with and without suction at the flap.

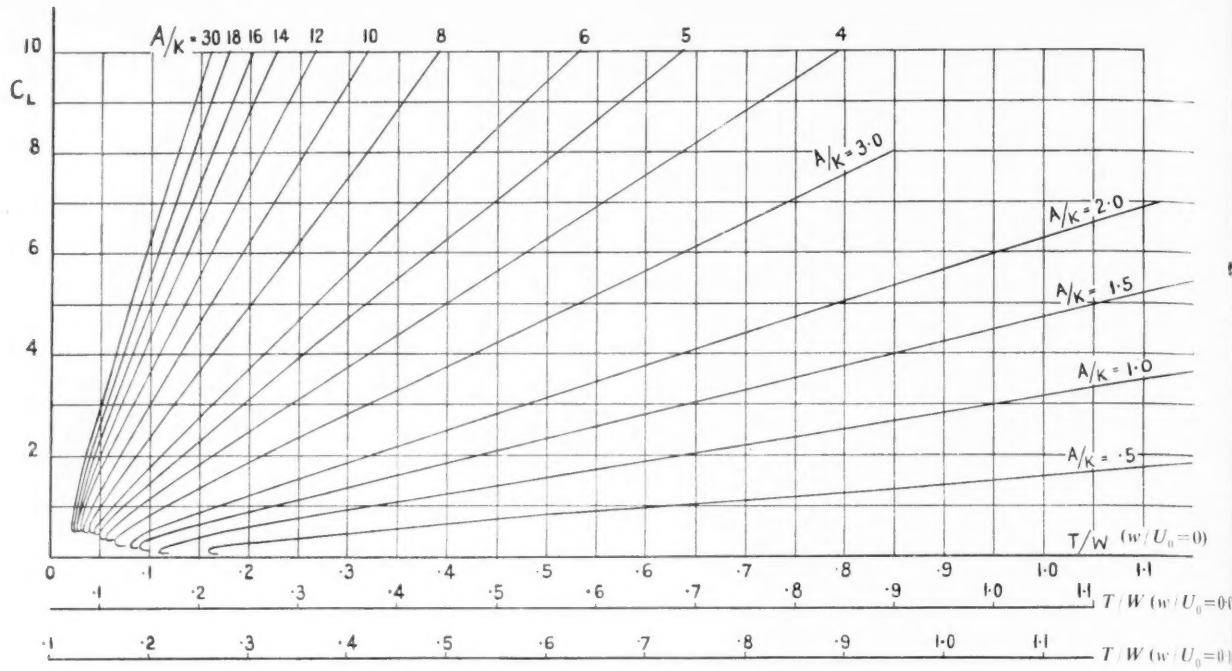


FIGURE 18. Relationship between take-off C_L and required thrust loading T/W with A/k as parameter at varying values of w/U_0 (w = rate of climb).

3. General Take-Off Considerations

Take-off length can be expressed with good approximation as

$$\text{constant} \times \frac{W^2}{C_L S T} .$$

The value of the constant is about 30 for conventional modern aircraft. For a given wing loading W/S, take-off length depends therefore on thrust loading T/W and C_L.

The minimum value of T/W required for horizontal flight is

$$(T/W)_{min} = D/L = C_L \frac{1}{\pi A/k} + C_{D_0}/C_L .$$

Rate of climb w at a given forward speed U_0 is

$$w = \left(\frac{T}{W} - \frac{D}{L}\right) U_0$$

or

$$\frac{w}{U_0} = \frac{T}{W} - \frac{D}{L}$$

or

$$\frac{T}{W} = \frac{w}{U_0} + \frac{D}{L}$$

$\frac{w}{U_0} + C_L \frac{1}{\pi A/k} + \frac{C_{D_0}}{C_L}$ = thrust loading required to provide a certain angle of climb for given values of C_L, C_{D_0} and A/k.

In Fig. 18 curves are plotted showing the relationship between take-off C_L and required thrust loading T/W with A/k as parameter at varying values of w/U_0.

This figure permits the necessary value of A/k and T/W to be extrapolated quickly to provide a desired climbing angle at a given value of C_L for take-off.

The graph indicates at a glance that for a certain value of w/U_0 and a relatively low thrust/weight ratio, very high values of A/k (*i.e.* high aspect ratio) are essential to make possible the use of high take-off lift coefficients.

Alternatively, if A/k is fixed by other considerations (wing weight) the thrust loading must be sufficiently high to enable the use of high take-off lift coefficients.

High take-off lift coefficients are not possible for aircraft of very low aspect ratios (supersonic aircraft). Such aircraft demand increasingly large thrust/weight ratios, even for relatively low lift coefficients at take-off.

In other words, the need for exceptionally high take-off lift coefficients decreases the more we proceed into the supersonic flight regime, and the need for high thrust loadings increases.

As the thrust/weight ratio approaches unity it becomes obviously more and more profitable to depart from the conventional method of taking off and to utilise thrust for lifting purposes in one form or another. Direct lift, as exemplified by the "Flying Bedstead," is not the only solution.

The design problem for take-off presents itself on subsonic aircraft often in the following form, except where particularly short take-off distances are needed at the expense of flight economy.

A given payload has to be carried over a given range at a certain cruising speed. Take-off distance under certain atmospheric conditions (I.C.A.N. or I.C.A.N.+ 30°) must not exceed a certain limit.

Cruise altitude is either stipulated or left, within certain limits, to choice.

Many types of engines suitable for transport aircraft (by-pass or turbo-prop engines) have the characteristic of reduced specific fuel consumption when throttled somewhat below maximum permissible r.p.m.

The choice is then between an engine of minimum size and weight, determined by cruising thrust without throttling, and a larger and somewhat heavier engine which is throttled during the cruise.

The first type of engine of lower weight will have a relatively high specific fuel consumption and a relatively low ratio of take-off thrust/cruising thrust, and a relatively high C_L will be required for take-off with a given wing loading.

The larger engine will be heavier, but its specific consumption will be a little lower. The aircraft will need a lower take-off C_L than in the first case.

The engine will provide a higher take-off/cruise thrust ratio and will be less sensitive to bleeding for energising the boundary layer system.

There will, therefore, be an optimum size of engine and a corresponding value of C_L for a given range and given take-off requirement where the combined weight of power plant and fuel becomes a minimum.

4. General Landing Considerations

In Figure 19 total drag is plotted over dynamic pressure q_0 resulting from its two components $D_i \propto (1/q_0)$ (induced drag) and $D_0 \propto q_0$ (parasite drag).

The flattest gliding angle $(L/D)_{max}$ occurs when $D_i = D_0$ (q_{opt}).

If the aircraft is flown at a speed with a corresponding value of $q_0 > q_{opt}$ stable conditions persist, *i.e.* an increase of speed increases resulting drag and *vice versa.*

However, if it is attempted to fly at a value of $q_0 < q_{opt}$ conditions are reversed, *i.e.* resulting drag decreases with increase of speed and increases when speed drops.

This phenomenon was first brought home to us in the early days of the slotted wing, when pilots attempted to approach at the high lift coefficients provided by the slot; they found that on applying elevators and increasing the angle of attack to flatten the glide path, rate of descent, instead of decreasing as in the conventional aeroplane, increased and the aircraft began to drop like the proverbial brick. An unconventional landing technique was gradually developed after the introduction of long stroke oleo legs, which were able to absorb increased rates of descent, while its final magnitude was controlled by the throttle.

For the following general considerations it has been assumed that the conventional relationship between approach and stalling speed exists (*i.e.* $U_0 = 1{\cdot}2$ or $1{\cdot}3\ U_{min}$), and that the lift coefficient during approach corresponds to the condition of maximum value of L/D, although it is appreciated that the usable range is not quite so sharply limited and that judicious use of the throttle and piloting skill make a certain intrusion into the "unstable" speed range possible.

Assuming a conventional method of landing, there

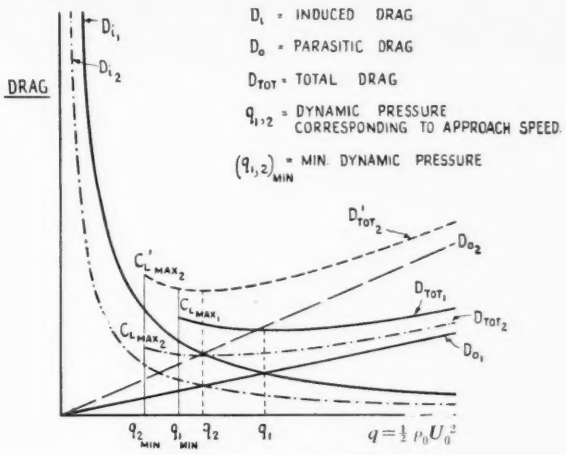


FIGURE 19. Generalised drag diagram.

are, therefore, two basic methods available when a reduction of the landing, or approach, speed of a given aircraft is considered.

If q_0 is to be reduced to q_0' by increasing C_L to $C_L' = C_L \cdot q_0/q_0'$, it is necessary either to reduce D_i (by increasing the span) or to increase D_0 by some form of additional drag which is proportional to dynamic pressure q_0 (*see* Fig. 19).

The first method is generally limited by consideration of wing structure weight, and the second method is limited by the available thrust to enable the pilot to climb away again in case of a baulked landing.

Again it is noticeable how imperative a departure from the conventional landing technique becomes for low aspect ratio aircraft by a better utilisation of thrust for landing.

One method which at first sight seems rather a paradoxical one (and probably not the best one) would consist in adding sufficient flat plate drag D_0, and "flying" the aircraft in, instead of "gliding," using thrust to overcome the added drag and thus avoiding excessively large gliding angles. Even where sufficient thrust is available, it becomes increasingly difficult to accommodate a sufficiently large drag area on a very clean aircraft.

These general considerations indicate, at any rate, that high-lift coefficients made possible by boundary layer and circulation control do not provide a unique solution to the problem of take-off and landing for all types of aircraft. A judicious application offers the best prospect.

SUMMARY

In judging the applicability of boundary layer or circulation control to existing aircraft, or potential aircraft of the future, the most obvious application is for deck landing aircraft and a new type of short-haul passenger aircraft characterised by very short take-off and landing runs of the order of 500 ft. over a 50 ft. obstacle, with correspondingly very low landing and take-off speeds, which would enable such aircraft to operate from small aerodromes.

All swept-wing aircraft, either in the subsonic or supersonic class, would benefit from the higher lifting capacity due to circulation control and the flap construction would be simpler and lighter than conventional high lift flaps.

The choice of maximum lift coefficients for take-off or landing depends on thrust and span loading of the design, and should be considered simultaneously with the power plant.

Blow-out is the simplest method from the engineering point of view, but is much heavier on mass-flow than either the suck or suck-blow method, where the installation may be a little more complicated.

Generally speaking, although wind tunnel research has by no means become redundant, the accent of importance has decisively shifted to the engineering and installation side. The practical success achieved with boundary layer control in the United States was due in the first place to the determined efforts to tackle the "plumbing" and to undertake flight research on suitable aircraft.

In both respects a good deal of leeway has to be made up in Great Britain.

PART II

LAMINARISATION THROUGH BOUNDARY LAYER CONTROL

1. Brief Historical Review of Theory and Experimental Research*

This review is confined to methods of stabilising a laminar boundary layer through suction, in contrast to other known methods utilising a favourable pressure gradient along the contour of an aerofoil, without or with final, removal of the boundary layer at a discontinuity of the contour through a suction slot. (Method developed by Goldstein and Griffith Section.)

1.1. THEORY

In 1936, Griffith and Meredith[2] gave the solution for the flat plate with uniform suction. This so-called asymptotic solution represented the condition eventually approached by the flow along the surface.

Several years later, the asymptotic solution for the flat plate with distributed suction was given independently by Holstein[3].

Estimates of the critical boundary layer Reynolds number for the profile in the asymptotic boundary layer by Bussmann and Münz[18], Pretsch[19], Hahnemann, Freeman and Finston[20] indicated much greater stability than the Blasius profile.

Approximate estimates of the length necessary for the boundary layer to develop the asymptotic condition were given by Schlichting[21] and Preston[22] and an exact solution by Iglisch[23].

Ulrich[24] determined the minimum ratio of suction of free-stream velocity which would preserve stability in the laminar boundary layer to infinite Reynolds numbers.

A criterion for calculating minimum suction velocities to maintain laminar flow is the stability theory by Tollmien and Schlichting[25] extended and improved by Lin[26] and others, and verified by Schubauer and Skramstad[27] and Liepmann[28].

Original theoretical attempts to deal with the general case of arbitrary distribution of pressure and suction velocity had indicated the limits of the Pohlhausen method. The general method suggested by Schlichting[29] is a development of the Pohlhausen method, using a wider range of profiles.

Schlichting's method and profiles were used by Burrows, Braslow and Tetervin[30] to calculate the critical boundary layer Reynolds numbers for these profiles and the suction quantities necessary to stabilise the laminar boundary layer over the surface of a 10 per cent. thick aerofoil at various Reynolds numbers.

Other methods dealing with distributed suction applied to a laminar boundary layer are due to Trilling[31], Head[32], Ringleb[33], Watson[34] and Thwaites[35].

Exact solutions of the boundary layer equations for certain distributions of suction and free-stream velocity to provide identical profiles (for a non-dimensional basis) at various points along the surface have been worked out by Thwaites[36], Schlichting[37], Bussmann[38] and Mangler[39].

1.2. EXPERIMENTAL WORK

1.2.1. *Great Britain*

Experiments by Perring and Diprose[40] in 1936, where suction was applied at closely spaced circumferential slots in the entry length of a 3 in. diameter circular pipe mounted in a wind tunnel, failed to prove the stabilising effect of suction predicted by Griffith.

Experimental work on distributed suction took place at Cambridge, J. M. Kay[41] working under the direction of Dr. Preston. Sintered bronze was used in these experiments, which verified the theoretical conclusions which had been reached by Griffith and Meredith ten years earlier.

Reynolds number of tests was low, and the suction quantities to maintain laminar flow were far in excess of those predicted by theory.

Flight experiments with distributed suction were made by Head[42] during 1949 and 1950 under the direction of Sir Bennett Melvill Jones. The test surface mounted beneath the fuselage of an Avro Anson was covered with Nylon. Reynolds numbers were restricted to below 4×10^6.

Both in the absence of a pressure gradient and with pressure distributions corresponding to those obtained on aerofoils, laminar flow was maintained through the use of suction, and the suction quantities agreed well with those predicted by the stability theory.

*Restricted to published and non-classified reports.

The present author has given particular attention to the engineering problems of boundary layer suction, that is the development of suitable surface construction and suitable ducting and pump arrangements; tests with large chord wings of realistic construction have been conducted at large Reynolds numbers, both in the wind tunnel and in free flight.

1.2.2. *Germany*

The first successful experiment was reported by Holstein[3] in 1940 on a 15 per cent. thick aerofoil at a Reynolds number of $1{\cdot}6 \times 10^6$. Laminar flow was maintained by means of suction through discrete slots over more than 90 per cent. of the chord.

These experiments were followed up by tests on a N.A.C.A. 0012/64 aerofoil having eight slots on the upper and seven slots on the lower surface. Laminar flow over more than 90 per cent. of the chord up to a Reynolds number of $3{\cdot}2 \times 10^6$ was obtained, but pump drag was found to be excessive.

1.2.3. *Switzerland*

Ackeret, Pfenninger and Ras[43] undertook experimental research in Switzerland during and after the Second World War, using slots to maintain laminar flow.

Pfenninger[44], since 1947, has continued experimental work in the U.S.A.

1.2.4. *United States*

Loftin and Burrows in the U.S.A. apparently did some initial experiments with slot suction as far back as 1939 and 1940.

Tests on a 3 ft. chord N.A.C.A.64A010 section with 41 suction slots in the Langley low turbulence tunnel were reported in 1952 (Burrows and Schwartzberg[45]). Full chord laminar flow was achieved for a chord Reynolds number of 10×10^6. At high Reynolds numbers premature transition occurred, due to minute surface irregularities and slight inaccuracies of slot shape.

Obviously the model chord was much too small for a representative demonstration of conditions in actual flight.

Braslow, Vincenti and Burrows[46] published in 1948 a preliminary investigation on the stabilising effect of distributed suction on laminar flow.

A model of 64A010 section was covered with blotting paper through which suction was applied with a certain initial success at a Reynolds number of about 4×10^6.

An improved model was then constructed having the same section, the outer porous surfaces being formed by a continuous sheet of sintered bronze which was smoothed by sanding. Originally, the model was not compartmented, and in order to prevent outflow at higher Reynolds numbers excessive suction had to be applied which thinned the boundary layer too much, so that transition occurred due to surface roughness.

After compartmenting and improving distribution of inflow, better results were obtained up to $R = 6 \div 7 \times 10^6$ (Burrows, Braslow and Tetervin[30]).

A great improvement resulted when the model was fitted with a sintered bronze surface of much finer grade than used previously. Virtually full-chord laminar flow was obtained up to a chord Reynolds number in the region of 24×10^6, by the use of very small suction quantities which were in good agreement with calculated values, except where excessive suction had to be applied to prevent outflow.

A drag reduction of 60 per cent. was obtained at $R = 19{\cdot}8 \times 10^6$, pump drag being taken into account (Braslow and Visconti[47]).

Dr. August Raspet[48] reported in December 1951 results of experiments obtained on a glider where suction was applied through rows of fine perforations on the upper surface of a test panel of the wing. Full-chord laminar flow was maintained over a wide range of lift coefficients. Drag reduction was 51 per cent. Reynolds number of tests was about 4×10^6.

2. Basic Aerodynamic Considerations. Considerations of Dynamic Similarity

A little confusion still exists on the proper definition of Reynolds number for a laminarised wing. If wings were infinitely smooth, the ordinary time-honoured chord Reynolds number would be a perfectly satisfactory basis of comparison. However, wings are not smooth and, since roughness has a very pronounced effect upon the stability of the laminar boundary layer, two wings of different chords are only comparable in regard to their behaviour with laminar flow provided that the ratio of the height of a surface roughness to wing chord c is the same for both wings.

Assuming a critical value of the displacement thickness $R_\delta{}^*$ at a certain position x/c of the chord of the wing with a certain method of boundary layer suction, the local displacement thickness at x will be $\delta_x{}^* = R_\delta{}^*/(U/\nu)$. There will be a critical value ε/δ^* which, when exceeded, will cause transition. It follows that

$$\varepsilon \propto R_\delta{}^*/(U_0/\nu) = \frac{R_\delta{}^* c}{R_c} \qquad \text{or} \qquad \varepsilon/c \propto \frac{R_\delta{}^*}{R_c}.$$

The following conclusions can thus be drawn:—

1. If the ratio $R_\delta{}^*/R_c$ is constant, permissible values of ε vary in the same ratio as the chords of model and wing. Actually, however, the critical value of $R_\delta{}^*$ critical may be higher in free atmosphere due to reduced turbulence and, hence, the ratio (ε/c) may be greater than for a wind tunnel model.
2. For a given type and size of roughness and a given value of $R_\delta{}^*$, the critical value of U_0/ν (unit Reynolds number) may be constant.

This statement is supported by the observation that the critical chord Reynolds number increased in direct proportion to wing chord for the same type of surface condition.

While it is obvious that chord is meaningless for the case of distributed suction, the same applies to a wing where suction is applied through discrete strips of porous material or slots. Such a wing of indefinite chord is best defined in terms of strip spacing l, the width s of a strip and ε as a measure of the roughness of the surface.

Supposing that at a given speed U_0, laminar flow can be obtained with a suction velocity v through the porous surface, then the suction quantity per strip $Q' = v \times s$, and a coefficient may be defined as $C_Q = Q'/s \times U_0 = v/U_0$.

Neglecting possible turbulence and compressibility effects, the principles of dynamic similarity show that for any wings that are similar to each other and for which the ratios s/l and ε/l are unchanged, the flow will be similar and, in particular, laminar, provided that the parameters $U_0 l/\nu$ and v/U_0 are the same.

It follows that C_Q per strip (as defined) is unaltered. It may be shown that R_δ* measured just in front of each strip is determined by the values of $U_0 l/\nu$ and C_Q and is, therefore, the same in both cases.

For a given standard of construction, surface roughness ε remains the same and, for similarity, the strip spacing and width must be kept constant although the chord may be increased. In other words, for a given value U_0/ν, any chord Reynolds number can be obtained.

It also follows that laminar flow may be obtained at higher speeds provided that the kinematic viscosity ν, and therefore the altitude of flight, is increased accordingly. This statement is strictly correct only while applied to a surface having spanwise sinks arranged at constant distance, l with ε/l = constant, and where the pressure gradient $\Delta p/l$ is constant. $R_l = U_0(l/\nu)$ is obviously the only significant criterion, and any chord Reynolds number R_c can be reached provided R_l is constant.

Consider two aerofoils at the same value of U_0/ν, both having the same standard of surface finish expressed by ε and the same strip spacing l.

The large aerofoil will have a greater number of strips, and the pressure gradient $dp/dx = f(x/c)$ will be less than for the smaller aerofoil. Therefore, if a given strip spacing l is found satisfactory on the aerofoil of smaller chord, it will certainly be satisfactory for a larger aerofoil at the same value of U_0/ν, and the larger aerofoil will, of course, operate at a greater chord Reynolds number.

The increase of speed with altitude at constant values of U_0/ν is considerable. A wing that is found satisfactory at a speed $U_0 = 200$ ft. per sec. at sea level should be equally satisfactory at 800 ft. per sec. at 45,000 ft., provided that there are no compressibility effects.

As an illustration, values for U_0/ν are plotted over flight Mach number M as a function of altitude H (Fig. 20). This figure shows the inter-relation between flight Reynolds number, chord, Mach number M, U_0/ν, a/ν and height. The type of aerofoil, especially the t/c ratio, fixes the Mach number. Assume $M = 0{\cdot}7$ and $U_0/\nu = 2 \times 10^6$, a value that does not make too exacting demand on surface finish. The height at which these flight conditions are fulfilled is $H = 30{,}000$ ft. The corresponding flight Reynolds number, assuming a 20 ft. chord wing, is $R_c = 40 \times 10^6$. If it were attempted to obtain the same chord Reynolds and Mach numbers on a wind tunnel model of, say, 8 ft. chord operating in an atmospheric low turbulence tunnel, the surface finish would have to be greatly improved to make possible the very high vaue of $U_0/\nu = 5 \times 10^6$.

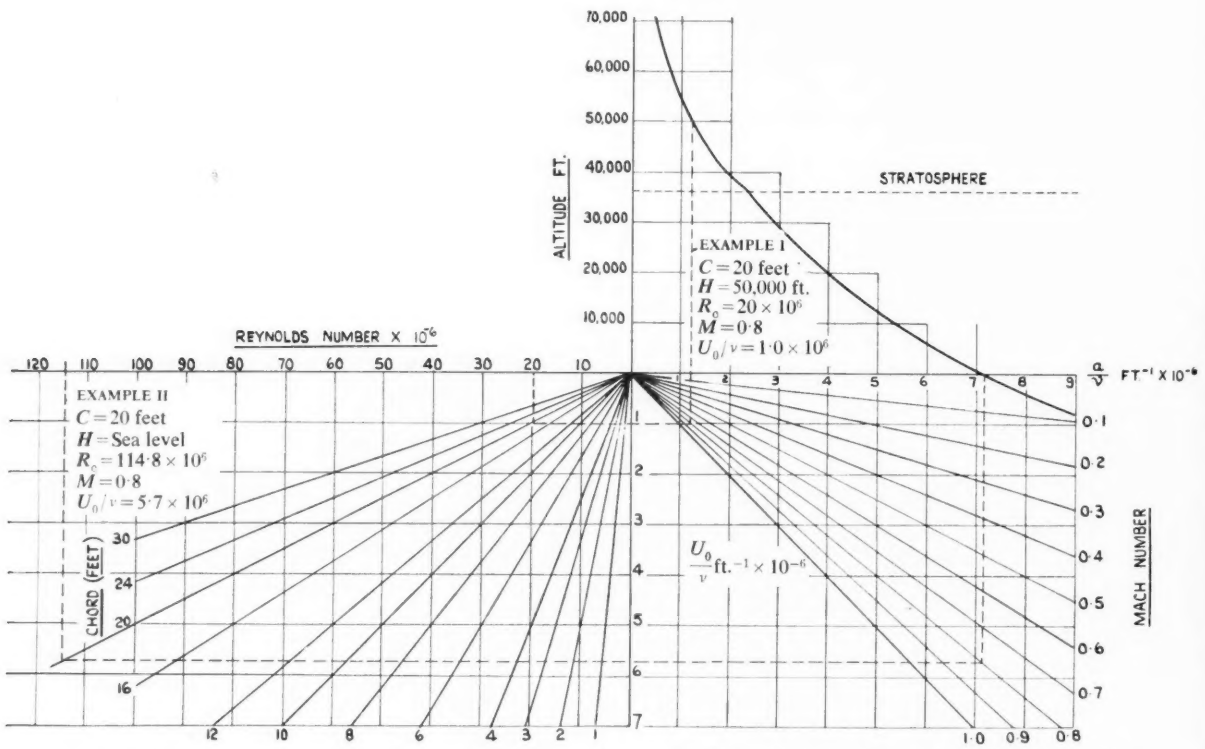


FIGURE 20. Connection between height, Mach number, U_0/ν (unit Reynolds number) and chord Reynolds number.

With a small chord model of, say, 3 ft., as used in Langley Field, the achievement of $R_c = 40 \times 10^6$ would necessitate excessively large values of U_0/ν, combined with reduced Mach numbers and considerable pressurisation.

This comparison may explain why those research workers who have studied laminar boundary layer flow on large chord models in atmospheric low turbulence tunnels, or in the free atmosphere, especially in the stratosphere, are considerably more optimistic about the practical possibilities of laminarisation than those who have conducted experiments with small chord models in pressurised wind tunnels. Fig. 21 shows an early model of a large chord wind tunnel model in realistic construction. This model had a chord of 8 ft. and was built by my company and tested at the National Physical Laboratory by Gregory and Walker. Fig. 22 shows observed transition (china clay method) at a value of $U_0/\nu = 1{\cdot}125 \times 10^6$. Actually, with this model, higher values of U_0/ν of the order of 2×10^6 with laminar flow back to 0·93 of the chord were obtained.

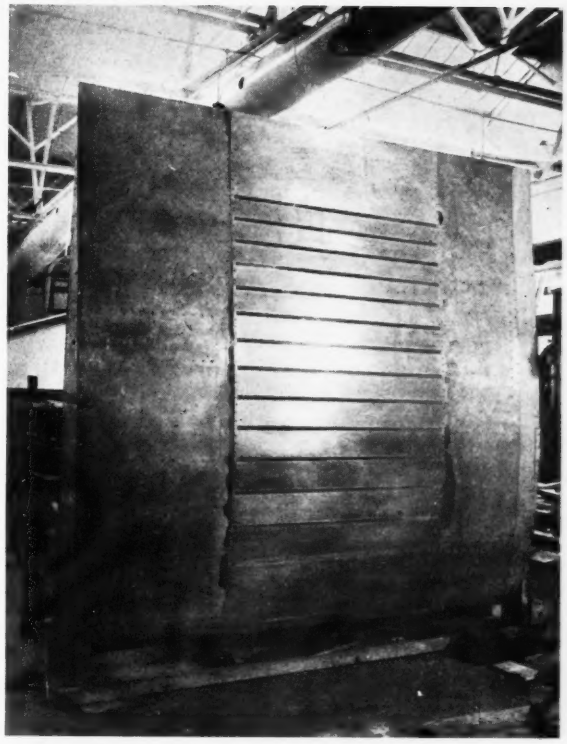


FIGURE 21. Handley Page wind tunnel model of wing.

2.1. METHODS OF LAMINARISATION

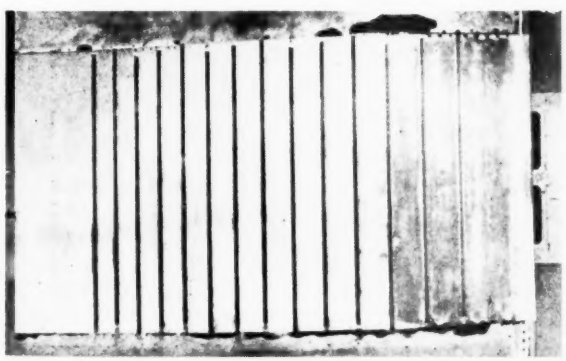
There are two basic methods which have been tried out, namely, distributed suction and suction through discrete sinks. The latter method was first used by Holstein and later by Pfenninger and by various research workers at N.A.C.A., and the sinks were usually constituted by narrow slots where a favourable sink effect was utilised. Distributed suction through a porous skin was used at the N.A.C.A. by Braslow and Visconti, and also by Sir Bennett Melvill Jones and Head, who used nylon and rolled gauze, heavily electroplated.

The argument put forward in favour of slots is that diffusor shaped slots enable a certain recovery of pressure, while with suction through porous material, a complete loss of dynamic pressure takes place. Slots, however, are difficult from the manufacturing point of view, especially the maintenance of the correct minute width. In model tests it was observed that they required frequent honing to keep the edges sharp.

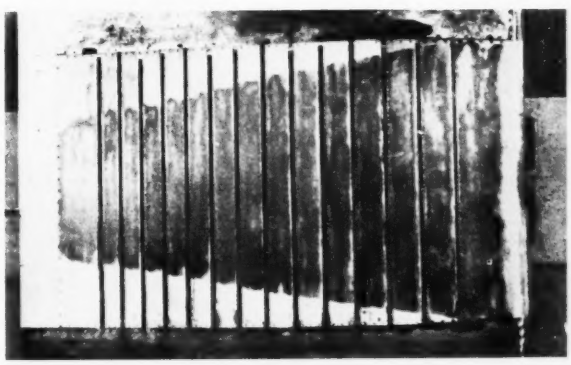
Distributed suction is the closest approach to the ideal theoretical method of an asymptotic boundary layer, but it is probably the most difficult method from a practical point of view.

The author introduced the method of sucking through discrete strips, which is a compromise between the two basic methods. These strips can be either of porous material or they can consist of rows of perforations.

In order to get a qualitative appraisal of the mechanism of laminarisation, and to define the essential design parameters, consider suction through two porous strips of width s separated in chordwise direction by distance l (Fig. 23).



(*a*) Transition on upper surface (without suction) at 0·28 chord.
$C_{D\ \text{SURFACE}}$ 0·003 22.



(*b*) Transition on upper surface (with suction) at 0·93 chord.
C_Q 0·000 27. $C_{D\ (\text{WAKE})}$ 0·000 78. $C_{D\ (\text{PUMP})}$ 0·000 37.
$C_{D\ \text{EFFECTIVE}}$ 0·001 15.

FIGURE 22. Observed transition with and without suction.

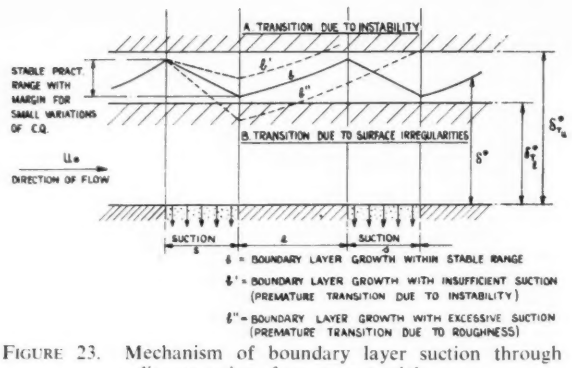

A TRANSITION DUE TO INSTABILITY
STABLE PRACT. RANGE WITH MARGIN FOR SMALL VARIATIONS OF C.Q.
B TRANSITION DUE TO SURFACE IRREGULARITIES
U_∞
DIRECTION OF FLOW
SUCTION
SUCTION
δ = BOUNDARY LAYER GROWTH WITHIN STABLE RANGE
δ' = BOUNDARY LAYER GROWTH WITH INSUFFICIENT SUCTION (PREMATURE TRANSITION DUE TO INSTABILITY)
δ'' = BOUNDARY LAYER GROWTH WITH EXCESSIVE SUCTION (PREMATURE TRANSITION DUE TO ROUGHNESS)

FIGURE 23. Mechanism of boundary layer suction through discrete strips of porous material.

The zig-zag line, b, represents the change of thickness of stable boundary layer. Reduction of boundary layer thickness occurs while boundary layer is withdrawn through the spanwise sink (negligible sink effect is assumed), followed by a build-up of thickness during the passage from one strip to the next. Upper and lower limits of the stable range are marked by $\delta_{Tu}{}^*$ (upper limit) and $\delta_{Tl}{}^*$ (lower limit). The upper limit is determined by stability considerations (Tollmien-Schlichting type instability) and the lower limit by transition due to surface roughness.

Dotted lines b and b'' indicate the effect of either under-sucking or over-sucking.

Over-sucking leads to transition due to roughness effect and insufficient suction leads to instability.

It is obvious that closing up the distance of strips (reduction of l) ensures stability with reduced local suction and less reduction of boundary layer thickness.

Hence, relatively greater surface roughness becomes tolerable for the same value of U/ν. If the strip spacing is reduced more and more, eventually the case of distributed suction is attained.

According to this, one should expect higher values of U/ν for a stable flow with distributed suction than for strip suction, surface roughness being the same in both cases. This seems to be confirmed by the experiments of Braslow and Visconti with a model of 64A010 aerofoil having a porous bronze skin, where the remarkably high value of $U/\nu = 5 \times 10^6$ was measured.

On examining boundary layer profile and thickness at the upstream and downstream edges of a suction strip, it will be found that profile I approaches more and more the ordinary Blasius-type as distance s between strips widens, while the profile II will generally be found more convex and almost identical with the profile of an asymptotic boundary layer (Fig. 24).

Reducing strip distance results in improvement at both ends of the range, profile I approaching the more stable shape of profile II, and boundary layer thickness becoming more evenly distributed.

According to Fig. 23, a narrow range of stable laminar flow is possible between an upper and a lower limit. The upper limit could be defined by a critical value of $(R_\delta{}^*)_{crit}$.

Comparison of $(R_\delta{}^*)_{crit}$ for a wing with suction applied through spanwise sinks and for one without suction, but having a zone of laminar flow due to a favourable pressure gradient, showed reasonably good agreement but the agreement between the values of $(R_\theta)_{crit}$ (Reynolds number (based on momentum thickness)) was much closer.

A very conservative value for $(R_\theta)_{crit}$ based on wind tunnel experiment is 1,250, including roughness effect. A tentative definition for the lower limit will be given later.

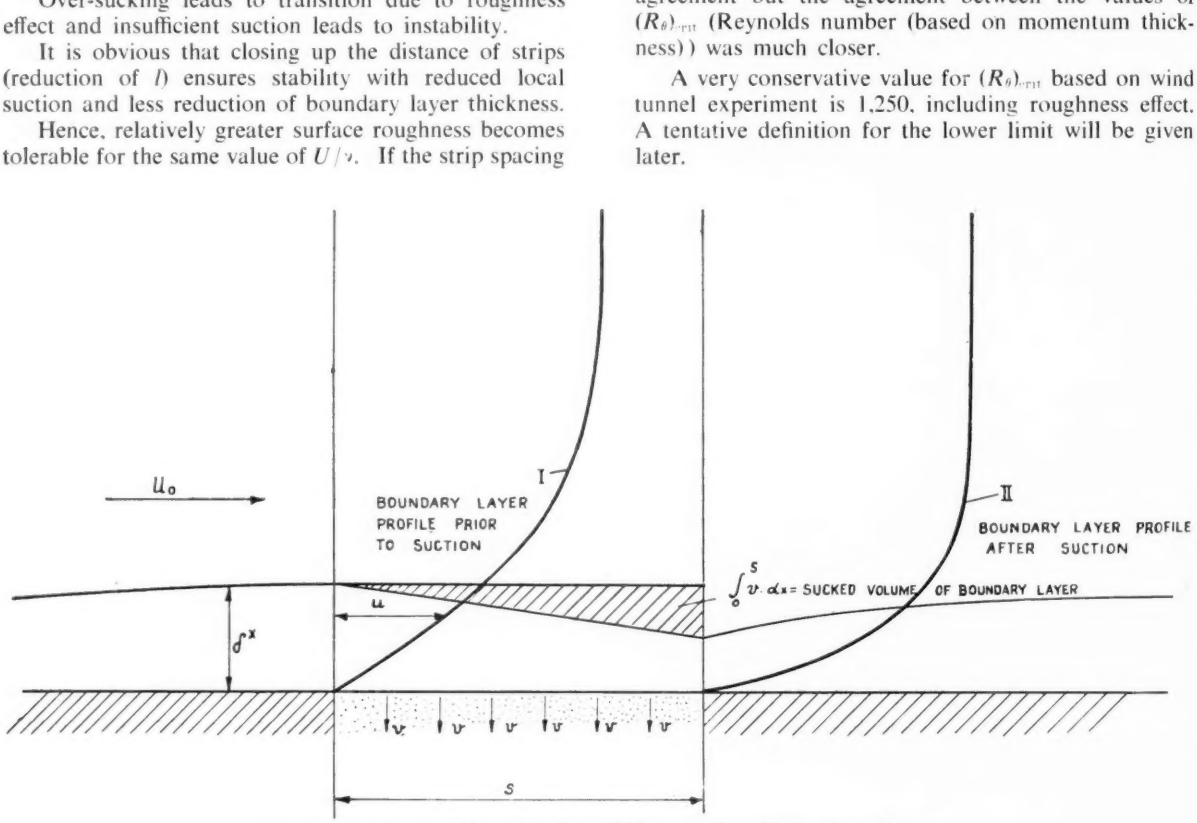


FIGURE 24. Change of boundary layer thickness and profile due to suction.

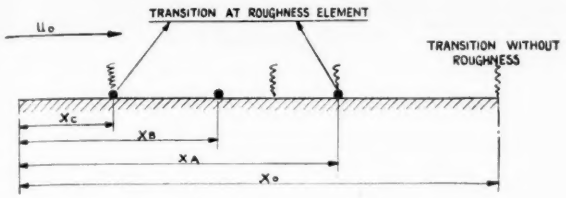


FIGURE 25. Transition at a roughness element.

2.2. THE EFFECT ON TRANSITION OF AN ISOLATED ROUGHNESS ELEMENT

The essential pre-requisite for laminarised surfaces is a surface finish that is compatible with the maintenance of a laminar boundary layer thinned down by suction.

Both upper and lower limits of the boundary layer thickness are critically affected by surface roughness or by an effect apparently akin to roughness but of a different nature, when suction is applied through perforations in the skin.

It has been known for some time that a roughness element of sub-critical height can move transition forward, *i.e.* towards the leading edge, without necessarily bringing it right up to the element itself. In other words, there is a kind of "infectious" transition and a type that might be termed "catastrophical" transition.

The two different types of transition were first disclosed by Fage's[49] experiments with roughness elements of various shapes in an air flow with favourable pressure gradient. The roughness element adds a further disturbance to those already present from the initial turbulence of the flow. Transition will occur earlier on a rough surface than on a smooth one, since with a larger disturbance less amplification is required.

Other investigations of the effect on transition of isolated surface excrescences in the boundary layer were made by Gregory and Walker[50] and by Loftin[51]. Klanfer and Owen[52] investigated the effect of isolated roughness on boundary layer transition for aerofoils with distributed suction, and the criterion used was a modification of the original criterion introduced by Schiller[53].

A recent analysis by Dryden[54], chiefly of Japanese experiments underaken by Tani, Hama and Mituisi[55] during the 1939-45 War, offers a good and transparent basis for estimating the effect of roughness on a boundary layer, especially a boundary layer with reduced thickness due to suction. An extension of Dryden's analysis is given in a paper on "Laminarisation through Boundary Layer Control" by the author[56].

It followed from this analysis that there are three characteristic regimes when a roughness element is gradually moved upstream from a downstream position of natural transition, *i.e.* the point where transition takes place on a flat smooth surface. As the element is moved upstream there will be a regime *A* where transition moves with the roughness element (Fig. 25).

Presently, transition begins to lag behind the roughness element (regime *B*), but finally transition catches up again with the element, and the two move together again (regime *C*).

If we designate with:—

δ_0^* = displacement thickness of laminar boundary layer at transition for a smooth surface plate

δ_T^* = transition displacement thickness for a rough plate

δ_ϵ^* = displacement thickness at the roughness element

the regimes *A*, *B*, *C* can be represented as shown in Fig. 26.

The following maxima were determined:—

$$\left(\frac{\varepsilon}{\delta_0^*}\right)_{max} = 0{\cdot}3375$$

$$\left(\frac{\delta_T^*}{\delta_0^*}\right)_{max} = 0{\cdot}676$$

It follows that

$$\left(\frac{\varepsilon}{\delta_T^*}\right)_{max} = \left(\frac{\varepsilon}{\delta_\epsilon^*}\right)_{max} = 0{\cdot}5$$

since

$$\delta_T^* = \delta_\epsilon^*$$

where transition reaches the roughness element.

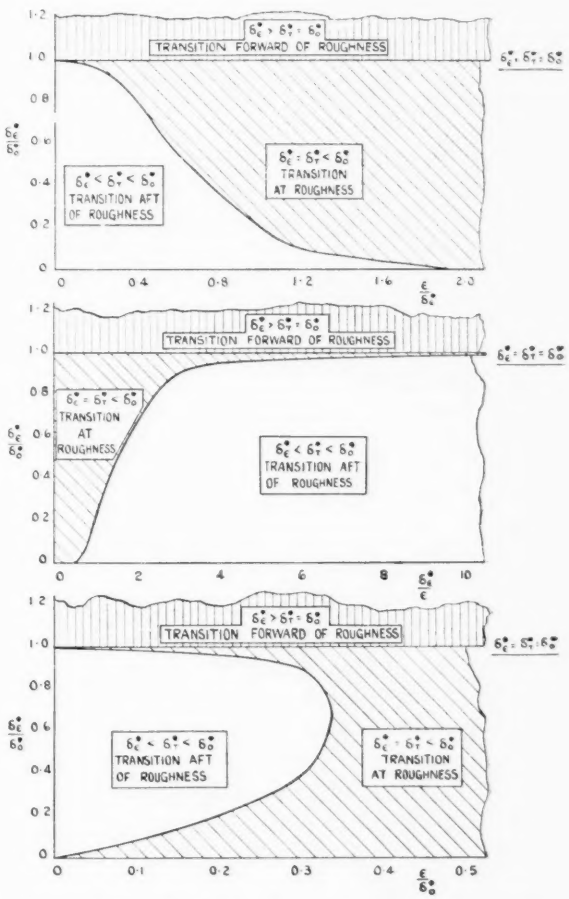


FIGURE 26. Effect of roughness on position of transition.

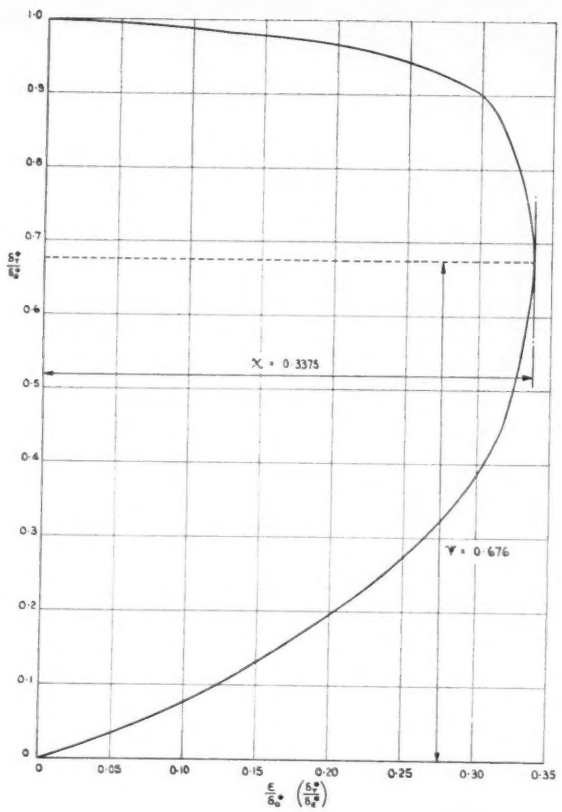


FIGURE 27. Variation of $\frac{\delta_T^*}{\delta_0^*}$ with $\frac{\varepsilon}{\delta_0^*}\left(\frac{\delta_T^*}{\delta_\varepsilon^*}\right)$.

In other words, the height of an isolated roughness element should be less than one half the displacement thickness of the boundary layer to avoid catastrophic transition at the roughness element.

Dr. Head[57] found in his flight experiments on an Avro Anson that, dependent on the type of velocity profile, laminar flow at low rates of distributed suction could be maintained if ε/δ^* was between 0·3 and 0·7.

Considering the case of a single suction strip, displacement thickness of the boundary layer just before suction is applied may be δ_u^* and δ_l^* after suction has been applied.

If transition is imminent:—

$$\delta_u^* = \delta_T^* \text{ (upper limit)}$$
$$\delta_l^* = \delta_\varepsilon^* \text{ (lower limit).}$$

In Fig. 27 the ratio δ_T^*/δ_0^* is plotted over

$$\frac{\varepsilon}{\delta_u^*}\left(\frac{\delta_T^*}{\delta_\varepsilon^*}\right).$$

A maximum value for δ_u^*/δ_l^* is reached when

$$\frac{\varepsilon}{\delta_0^*}\frac{\delta_T^*}{\delta_\varepsilon^*} = \frac{\varepsilon}{R_{\delta_0}^*}\left(\frac{U}{\nu}\right)\frac{\delta_u^*}{\delta_l^*} = 0{\cdot}338$$

or the minimum value of the ratio of the displacement thicknesses after and before applying suction becomes

$$\left(\frac{\delta_l^*}{\delta_u^*}\right)_{min} = \frac{1}{0{\cdot}338}\frac{\varepsilon}{R_{\delta_0}^*}\left(\frac{U}{\nu}\right). \quad . \quad (1)$$

This ratio can also be expressed in terms of momentum thickness θ.

Assuming that the profile of the boundary layer when reaching the suction strip is of the Blasius-type and that the profile after suction corresponds to an asymptotic boundary layer,

$$\frac{\delta^*}{\theta} = 2{\cdot}62 \text{ (Blasius profile)}$$

$$\frac{\delta^*}{\theta} = 1{\cdot}96 \text{ (for the profile of an asymptotic boundary layer)}$$

or
$$\left(\frac{\theta_l}{\theta_u}\right)_{min} = 1{\cdot}34\left(\frac{\delta_l^*}{\delta_u^*}\right)_{min}.$$

From wind tunnel tests with a model having suction strips, the following values were derived from measured boundary layer profiles:

$$\frac{\delta_u^*}{\theta_u} = 2.35$$

$$\frac{\delta_l^*}{\theta_0} = 2{\cdot}01$$

$$\left(\frac{\theta_l}{\theta_u}\right)_{min} = 1{\cdot}17\left(\frac{\delta_l^*}{\delta_u^*}\right).$$

A suggested mean value is

$$\left(\frac{\theta_l}{\theta_u}\right)_{min} = 1{\cdot}25\frac{\delta_l^*}{\delta_u^*} \quad . \quad . \quad . \quad (2)$$

Substituting the value for δ_l^*/δ_u^* from (2) into equation (1),

$$\left(\frac{\theta_l}{\theta_u}\right)_{min} = 3{\cdot}7\ \frac{\varepsilon}{R_{\delta_0}^*}\frac{U}{\nu}. \quad . \quad (3)$$

Assuming $R_{\delta_0}{}^*{}_{crit} = 4{,}500$ (equivalent to $(R_\theta)_{crit} = 1{,}250$)

$$\left(\frac{\theta_l}{\theta_u}\right)_{min} = \varepsilon\ \frac{U}{\nu}823 \times 10^6. \quad . \quad . \quad (4)$$

This simple relation, represented in Fig. 28, permits several important conclusions to be drawn for the design of suction surfaces.

Obviously, the ratio θ_l/θ_u cannot become greater than unity. (Case of distributed suction.) From $\theta_l/\theta_u = 1$ follows the greatest value of U/ν which can be expected for a given surface finish defined by ε, or alternatively, maximum permissible roughness to attain a certain value of U/ν can be specified. The linearity for $\theta_l/\theta_u = f(U/\nu, \varepsilon)$ only holds for a certain range of θ_l/θ_u.

It can be expected that the straight lines will bend to the right as the case of distributed suction is approached. Higher values of U/ν may then be expected with a given surface finish, since the value of $(R_\theta)_{crit}$ will become greater than the conservative value derived from experiments with discrete suction sinks, spaced at relatively wide intervals.

If a wing with discrete sinks was found satisfactory at a certain value of $(U/\nu)_1$, one can extrapolate how the ratio θ_l/θ_u has to be increased by reducing local suction quantities, *i.e.* by decreasing the chordwise distance between sinks, in order to obtain a value of $(U/\nu)_2 > (U/\nu)_1$ with the same surface finish (*see* Fig. 29).

The Reynolds number $\varepsilon U/\nu$ emerges, thus, as the most important criterion for suction surfaces. Design procedure of a suction wing with discrete sinks is as follows:

Starting from the leading edge, growth of θ/c with x/c is calculated for the unsucked boundary layer. A curve is drawn which represents the critical value of θ/c along the chord according to the local values of U/ν.

When, or preferably before, the boundary layer growth curve approaches the top limit of (θ/c) the first spanwise sink must be arranged (Fig. 30).

Suction is now applied through the sink until a permissible lower value of θ_1 is approached. A relationship between the suction coefficient C_Q and the ratio θ_1/θ_u can be derived with the assumption that the boundary layer profile II (after suction) obtains from the profile I (prior to suction) by cutting off the lower tip of profile I. In other words $(\theta_1/c)/(\theta_u/c) = f(C_Q)/(\theta_u/c)$ (Fig. 31).

Starting from the new value (θ_1/c) the calculation of the boundary layer growth is repeated in the same manner until the growth curve approaches again a critical value of θ_u/c.

This process continues until the trailing edge is reached.

The method has given reasonably good agreement with experimental observations (Fig. 32).

It is apparent from the foregoing, for either method of suction (distributed or strip suction), that requirements relating to surface finish become more stringent as the value of U/ν increases. This applies particularly to the forward part of the aerofoil where the boundary layer thickness is very thin. Here, a very smooth skin is essential and it would seem advisable to rely on the natural laminar state of the boundary layer for a certain distance, since it is generally easier to obtain a smooth impervious surface than a porous one. I do not know whether the idea has ever been investigated to de-sensitise the thin boundary layer which is formed at the nose of the aerofoil by letting air seep through a porous surface, thus thickening up the local boundary layer. (Controlled and distributed outflow.)

The desired range of cruising lift coefficient determines the progress of the laminar front in chordwise direction and it is worth observing that this progress decreases or reverses as the lift coefficients increase

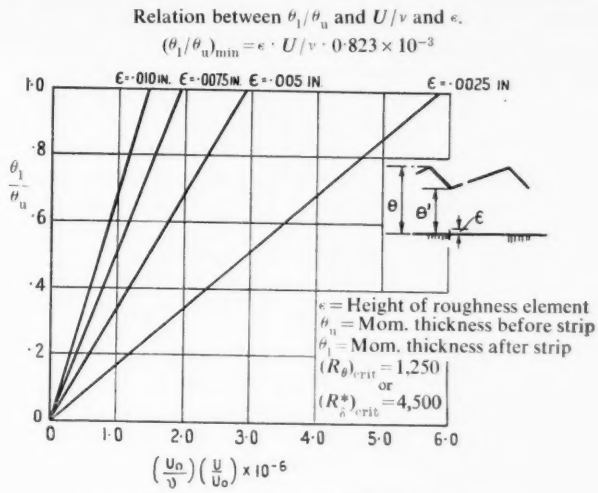


FIGURE 28. Relation between θ_1/θ_u and U/ν and ϵ.

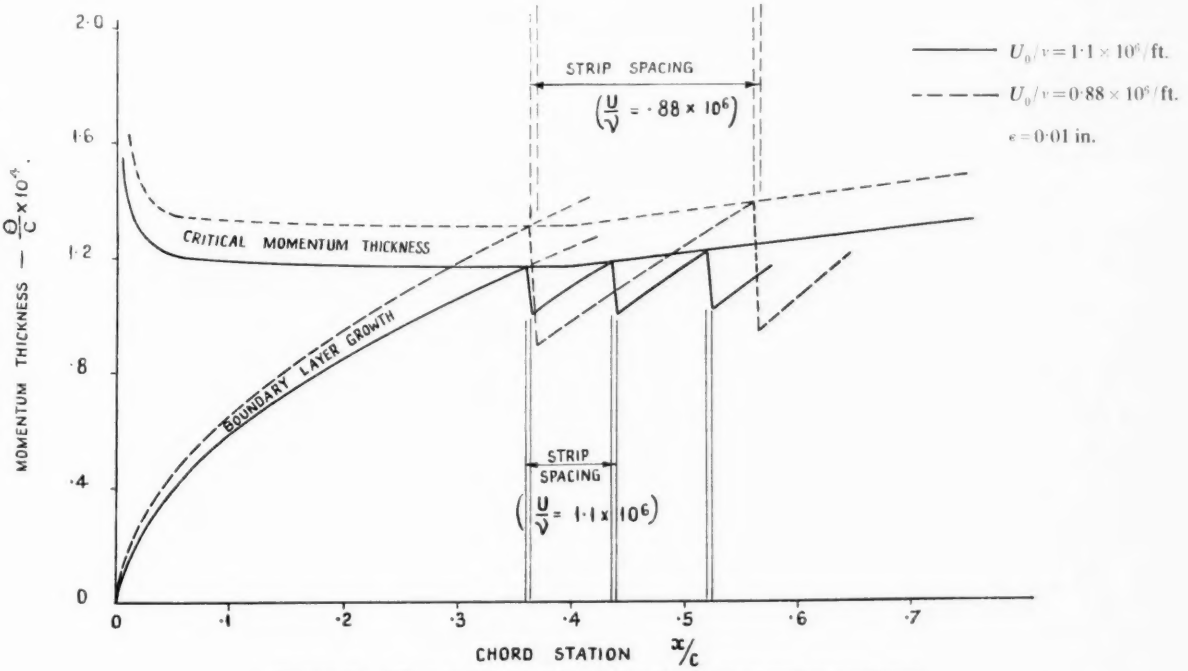

STRIP SPACING
$\left(\frac{U}{\nu} = \cdot 88 \times 10^6\right)$
$U_0/\nu = 1{\cdot}1 \times 10^6$/ft.
$U_0/\nu = 0{\cdot}88 \times 10^6$/ft.
ε = 0·01 in.
CRITICAL MOMENTUM THICKNESS
BOUNDARY LAYER GROWTH
STRIP SPACING
$\left(\frac{U}{\nu} = 1{\cdot}1 \times 10^6\right)$
MOMENTUM THICKNESS — $\frac{\theta}{c} \times 10^4$
CHORD STATION $\frac{x}{c}$

FIGURE 29. Effect of U_0/ν on strip spacing for the same surface roughness.

beyond the value of C_L for which the section is designed to have a favourable pressure gradient.

The natural laminar front should overlap the sucked portion of the surface by a reasonable margin, since the fringe of the natural laminar front is already in an infectious state tending toward instability. Aerofoil contours should be carefully designed and velocity distrition checked to make sure that there are no isolated kinks or adverse pressure gradients near the leading edge within the desired range of lift coefficients. From a practical point of view, distributed suction is more difficult to achieve with a high standard of surface finish than with suction at intervals.

The ultimate choice would seem to be a compromise, *i.e.* suction through closely spaced spanwise sinks.

Roughness defined in the foregoing by the value of a roughness element includes inherent and acquired surface blemishes; that is, those due to manufacture and fly and dust accretion.

Means exist, either to protect the nose of the wing during take-off and climb to cruising altitude, or to remove collected flies in flight.

Dust is collected generally only when the aircraft stands in the hangar. Special dust covers for the wings will become necessary in service to prevent dust collection.

2.3. OTHER EFFECTS LEADING TO TRANSITION

Roughness, although probably the most serious, is not the only cause which may destabilise a laminar boundary layer.

Transition may be caused also by: excessive surface waviness, local outflow, and the occurrence of a secondary flow in the boundary layer perpendicular to the streamlines in the outer flow.

2.3.1. *Waviness*

Waviness is not synonymous with roughness but qualitatively similar in effect.

It has been observed that a laminar boundary layer stabilised by suction will be found more inclined to overcome small adverse pressure gradients due to waviness than actual surface excrescences.

If waviness is not greater than about 0·003 in. within a 3 in. curvature gauge, there should be no trouble up to values of $U/\nu \leqq 3 \times 10^6$.

2.3.2. *Outflow*

Outflow can happen and destabilise the laminar flow when the pressure drop through the porous surface is smaller than the outer pressure gradient in spanwise or chordwise direction.

On wind tunnel models porous materials like blotting paper, felt, Porosint, and so on, of sufficient flow resistance have been used with success, but on actual wings different solutions have to be found.

2.3.3. *Instability Due to Secondary Flow Normal to the Outer Streamlines*

W. E. Gray, of the Royal Air Force, discovered in 1952 by visual observation of transition on a number of swept-back wings and fins that sweep angle had a profound de-stabilising effect on the laminar boundary layer.

Subsequent observations indicated that the resulting instability started in the form of chordwise striations

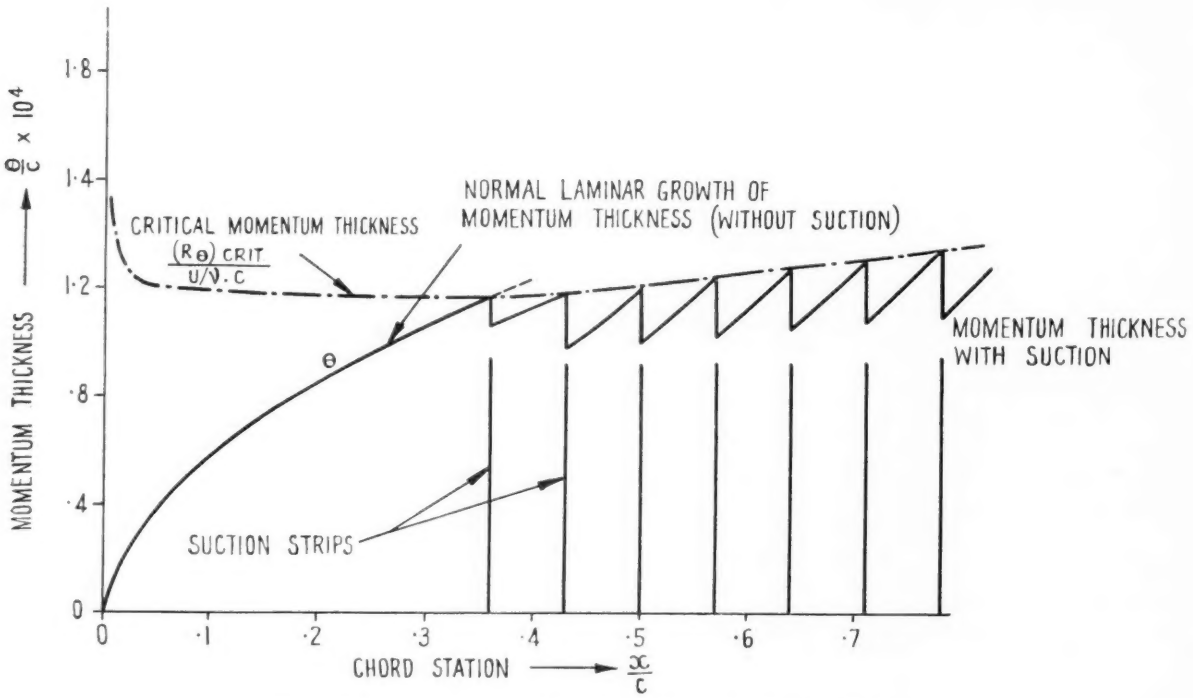


FIGURE 30. Aerodynamic design for a suction surface with discrete strips.

spaced at distinct intervals. These striations were similar in appearance to the vortex trails which appear in the flow over a concave surface. (Görtler vortices.)

The instability was associated by Squire, Owen and Randall and Stuart, with a secondary flow in the boundary layer caused by the curvature, in planes tangential to the wing surface, of the stream lines just outside the layer.

The velocity profile in this secondary flow, measured normal to the wing surface in a plane perpendicular to the stream lines in the outer flow, contained a point of inflexion indicating that dynamic instability of the secondary flow could be expected to develop above a certain Reynolds number, ultimately leading to the formation of vortices with axes about parallel to the outer flow direction (Fig. 33).

Ascribing to the secondary flow a Reynolds number based on the maximum velocity $|U_\beta|_{\max}$ (normal to the streamline just outside the boundary layer) and the depth δ of the profile,

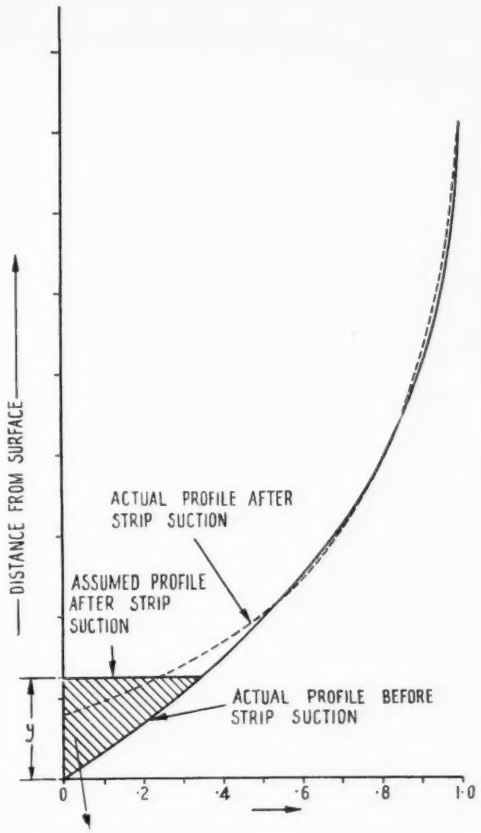
$$\chi = \frac{|U_\beta|_{\max} \cdot \delta}{\nu}$$

the critical value for χ was found in agreement with wind tunnel and flight observations to be of the order of 125. It is significant that critical Reynolds numbers of this order of magnitude have been observed in tests on laminar wakes, which also exhibit a dynamic instability.

The distribution of χ over the surface of an infinite swept-back wing was calculated by a method similar to the one developed by Squire for finding the heat transfer from an aerofoil in two-dimensional flow.

The calculations by Owen and Randall indicated that $\chi/R^{\frac{1}{2}}$ reaches a maximum near the leading edge and then decreases slowly farther downstream, so that we should expect the first signs of instability to appear near the leading edge.

A study of the variation of sweepback showed that



$$\frac{Q}{U} = \int_0^y \left(\frac{u}{U}\right) dy \qquad \frac{\theta_1}{\theta_{1_0}} = 1 - \frac{1}{\theta_{1_0}} \int_0^y \frac{u}{U}\left(1 - \frac{u}{U}\right) dy = 1 - \frac{\int_0^y \frac{u}{U}\left(1 - \frac{u}{U}\right) dy}{\int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy}$$

$$C_Q = \frac{1}{c} \int_0^y \left(\frac{u}{U}\right) dy$$

FIGURE 31. Actual and assumed profile after strip suction.

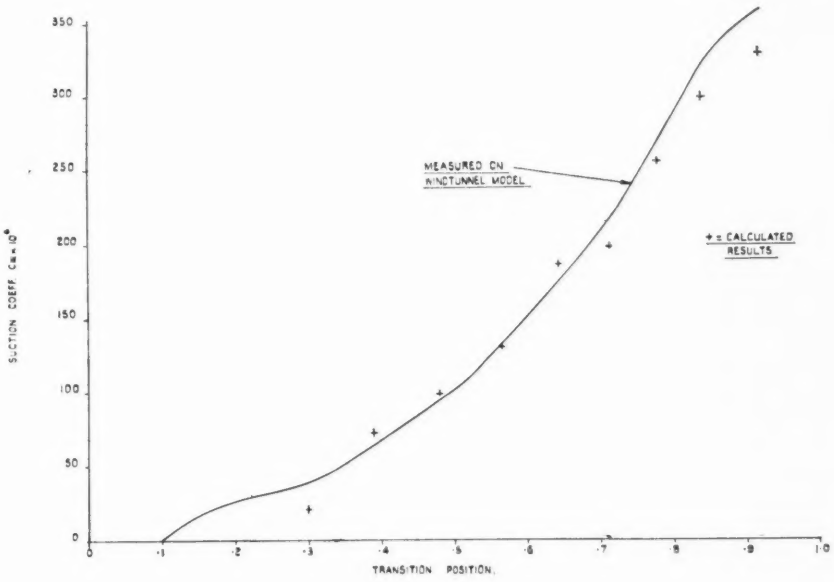


FIGURE 32. Comparison between calculated and measured suction coefficients.

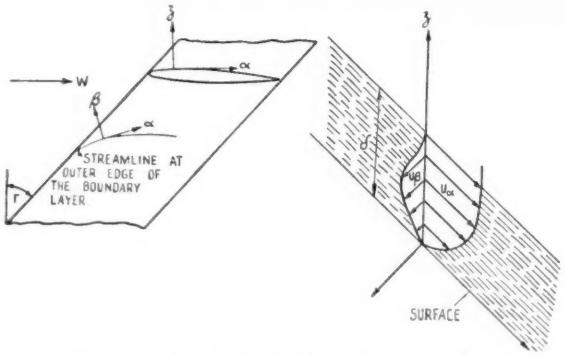


FIGURE 33. Boundary layer profile in plane normal to streamlines outside the boundary layer.

$\chi/R^{\frac{1}{2}}$ increases with angle of sweepback and reaches a flat maximum at 45° and then falls again at higher angles of sweepback. It is obvious that $\chi/R^{\frac{1}{2}}$ must be zero when the angle of sweepback is 90° (Fig. 34).

Increase of thickness/chord ratio throughout the range of sweepback was found to increase the value of $\chi/R^{\frac{1}{2}}$ and *vice versa.*

The theoretical explanation was checked by wind tunnel experiments at the R.A.E. on a wing of 4 ft. chord and 15 per cent. thickness/chord ratio, both measured in a plane normal to the leading edge. The tests were made at zero incidences. Similar experiments with similar results were made at the N.P.L. on a wing of 2·5 ft. chord, 30 per cent. thickness/chord ratio with sections of the Griffith suction type.

It was found in the tests that, for a given angle of sweepback, three distinct changes in the boundary layer could be detected as the wind speed was increased.

At low speeds, transition remained far back from the leading edge at the position appropriate to zero sweepback (approximately 65 per cent. chord). At a certain fairly well defined speed, striations in the china clay, starting near the leading edge, became visible without any immediate effect on transition.

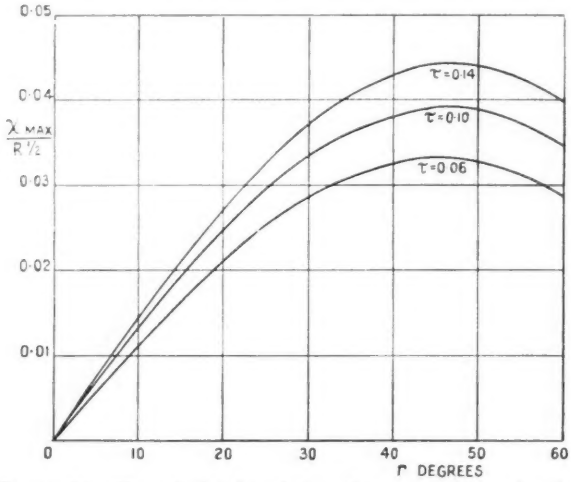


FIGURE 34. The calculated variation of χ max. with angle of sweepback for thickness/chord ratios of 6 per cent., 10 per cent. and 14 per cent. for the R.A.E. 102 skin.

With a small increase in speed, the striation pattern became more intense and transition began to move forward.

As the speed increased further, the forward movement of transition continued until the leading edge was reached.

The rapidity with which the forward movement took place depended on the angle of sweepback: the movement was gradual at small angles of sweep and sudden at large angles.

The important conclusion is, therefore, either to avoid sweepback or limit it to angles $<20°$, or to apply very pronounced sweepback of the order of 60° to 70° combined with very low thickness/chord ratios.

The first type of wing would be applicable for medium subsonic Mach numbers $M<0·8$, the latter type, typical for supersonic aircraft, would indicate the possibility to laminarise such wings for flight at high subsonic Mach numbers and great altitude.

Since $\chi/R^{\frac{1}{2}}$ and χ assume critical values for given angles of sweepback and t/c ratio, a critical Reynolds number can be derived which is defined by U_0/ν and the chord c in stream direction. For a given chord c there will be, therefore, a critical value of U_0/ν.

As shown before (Fig. 20), high Mach numbers are correlated to low values of U_0/ν if the altitude is great enough.

The effect of sweep angle on transition becomes, therefore, less critical (for a desired Mach number) as the altitude of flight increases. There is a parallel to the effect of surface roughness on transition.

If strategic conditions should permit a substantial percentage of the range to be covered at high subsonic speed, a greater fuel economy might result in this way.

An alternative to either a straight or slightly swept-back wing might be a wing having a straight or slightly swept-forward leading edge with swept-forward isobars.

The isobar sweep, strongest over the inner part of the wing where t/c is largest, results from using wing sections at the root having the point of maximum suction very far aft in chordwise direction with the suction position moving gradually more towards the leading edge over the outer and thinner position of the wings, where less sweep of the isobars is necessary for the same critical Mach numbers.

It might be possible by this method to obtain reasonably high subsonic Mach numbers due to isobar sweep and to avoid the destabilising effect experienced at the leading edge of a swept-back wing.

3. Some Practical Design Considerations

3.1. DESIGN OF SUCTION SURFACES

Essential requirements, according to the foregoing, are:—

(*i*) Local surface roughness should be less than half local boundary layer displacement thickness.

(*ii*) Waviness should be $<0·003$ in. within a 3 in. curvature gauge (to satisfy flight at $U_0/\nu \leqq 2·5\times10^6$).

(*iii*) Pressure drop through the suction surface should be greater than pressure variations in spanwise or chordwise direction on the surface (to prevent outflow).

(*iv*) Reynolds number $\chi = /U_\beta/_{\max} \cdot \delta/\nu \leqq 125$ where U_β is component of flow velocity perpendicular to outer streamline. δ = boundary layer thickness.

In addition to these aerodynamic desiderata there are the additional requirements that the suction surface should not increase appreciably the weight of the wing structure, and it should not require excessively costly or difficult maintenance.

These are, indeed, formidable engineering requirements.

However, experience in many fields of engineering has proved that the major step towards the solution of any technical problem is clearly and exactly to recognise and analyse the important parameters and to specify requirements accordingly.

It will be understood that I cannot disclose in this paper the latest ideas or methods of how to achieve an engineering solution for a suction surface. However, some basic principles are disclosed in recently published British Patent Specifications (No. 687723 (Head) and No. 713426 (Lachmann and Handley Page Ltd.)). The first Specification (687723) discloses a method of constructing a suction surface suitable for the application of distributed suction (Fig. 35). It comprises an inner skin of non-porous material (sheet) and an outer skin of porous sheet material, separated by an interspace and inter-connected by a lattice of web members extending across the interspace and subdividing it into a number of non-intercommunicating cells, each of which communicates through an opening in the inner skin with a chamber, which is common to a number of such cells and to which suction can be applied.

The opening in the inner skin may be provided with a non-return valve and has an effective area and discharge coefficient, so that when flow takes place through the pores of the outer skin, and through the opening, the loss of head through the latter is considerably greater than through the porous outer skin.

The elastic moduli and gauge thicknesses of the materials used are such that under any condition of loading to which the skin may be subjected the outer porous skin is lightly loaded relative to the inner skin and cell-forming lattice.

The non-return valves are visualised in the form of simple flaps of flexible non-porous material.

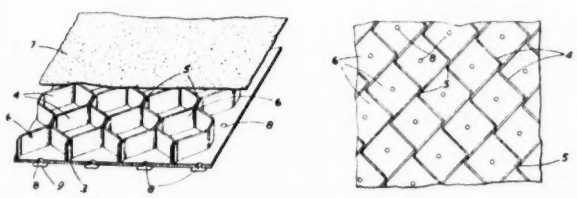


FIGURE 35. Method of constructing a skin surface according to British Patent Specification 687723 (*Head*).

The necessity for non-return valves or equivalent devices arises from the necessity to avoid strong in- and outflow tendencies with suction inoperative. Outflow due to chordwise pressure difference causes thickening of the boundary layer and substantial increase of profile drag and premature stalling in such flight conditions where no suction is applied.

A similar method for the application of distributed suction is disclosed in Spec.713426 (Fig. 36) where the surface comprises an inner load carrying sheet having perforated recessed areas separated by exterior ridges milled from the inner sheet and integral with it, and a continuous substantially unstressed porous outer surface stretched over the inner skin and attached to the ridges.

The structural element comprises a corrugated sheet member and a second skin. Thus ducts are formed for the air sucked from the boundary layer within this sandwich panel without the expenditure of non-stressed material.

Further applications of the principle of integrating a non-stressed porous member to a skin panel incorporating the suction ducts as load-carrying members are illustrated in Fig. 37.

These applications refer to the method of suction through discrete suction strips of porous material spaced at chordwise intervals with portions of impervious skin in between.

Porous strips are inserted in the outer skin by means of a sliding fit, with the shear flow in the outer skin by-passing the porous strip so as to leave it substantially unstressed.

A form of construction is also disclosed where closely spaced perforations in the outer continuous skin replace the inserted porous strips, and where an element of suitable diffusing material could be inserted below the perforations in case the pressure drop through the perforations was insufficient to prevent local in- and outflow through the holes in the outer skin.

Both patents quoted were applied for in 1950. Broadly speaking, the principles disclosed have been found feasible methods for the construction of suction surfaces. However, a good deal of effort has been devoted since then to translating these basic methods into a suitable form of "hardware."

The high degree of surface smoothness is difficult to achieve and requires a good deal of very accurate workmanship, and often a lot of "elbow-grease," if the application to orthodox methods of wing construction is attempted.

I feel, therefore, that one has to look forward to novel forms of construction. Such unconventional methods may lead to the use of prefabricated sheaths in plastic material incorporating a suitable type of porosity, bonded to the upper surface of a metal panel in sandwich construction, utilising stress carrying members as ducts.

The double skin sandwich panel has the added advantage of providing a firm outer surface well supported by the internal duct members and with reduced tendency to develop surface waviness.

To reap the greatest benefit from laminarisation, the affected surface should be as large as possible and there

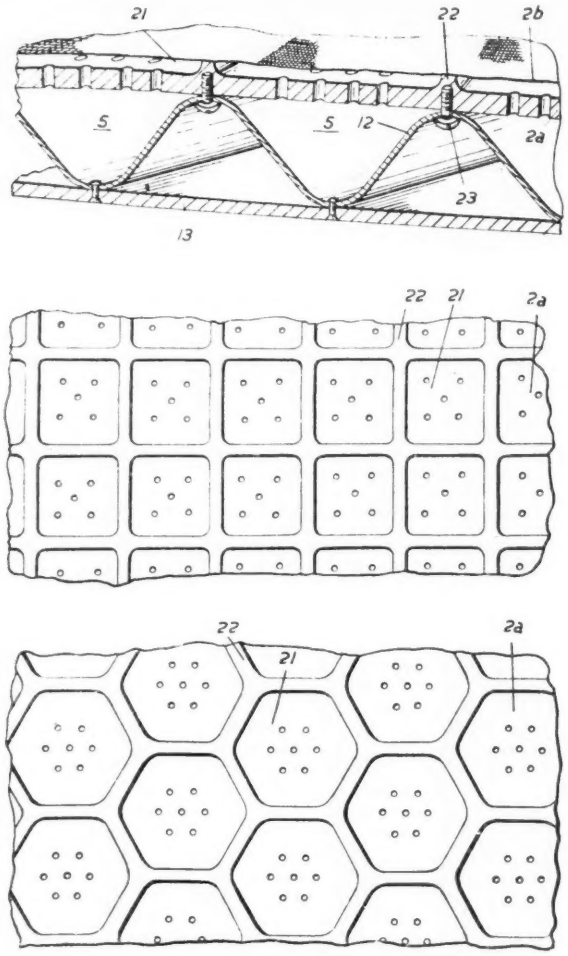


FIGURE 36. Method for distributed suction according to British Patent Specification 713426.

seems to be no fundamental reason why, ultimately, the fuselage surface should not be laminarised as well as the wings.

The wing surface should be smooth over the greatest possible spanwise and chordwise extension. Engine and undercarriage doors should be avoided, if possible, also spanwise and chordwise joints, inspection doors and gaps due to ailerons and flaps. This may lead to the replacing of standard ailerons and flaps by some form of circulation control.

3.2. SUCTION PLANT AND DUCTING

The remaining wake drag of a fully laminarised wing is very small and can be reduced to zero by accelerating the boundary layer somewhat above free stream velocity.

The main effort is concentrated in pump work which results from re-energising the boundary layer to free stream velocity (or above) and from overcoming external suction on the wing and making good duct and pressure losses encountered in sucking the boundary layer through the porous surface.

A general analysis of the various components making up pump power is given in Fig. 38.

The primary item is re-acceleration of the boundary layer. Since the outside suction pressure is not equally distributed over the wing it obviously pays to have a graded suction pressure multi-stage pump. Detailed investigations have indicated that the saving due to using a multi-duct system with graded suction pressure would be of the order of 15-18 per cent., and that it would not pay to apply more than three stages.

It is obvious that the problem of grading flow resistance through the porous surfaces to avoid outflow is simpler with discrete porous strips than with distributed suction.

The suction-compressors themselves should offer no particular design problem since the pressure ratio required is only of the order of 2·5-3·0. A subsonic axial flow compressor of three stages, aspirating air simultaneously at each stage from different pressure levels, would seem to be a practical solution.

It seems fairly obvious that the boundary layer air should not be passed as working fluid through the turbine but the latter aspirating ram air through its compressor should drive a separate boundary layer compressor.

Preferably the compressors should be driven by direct power take-off (shaft drive) from the turbine rotors, or by separate gas turbines fed by the main engines or, generally speaking, by gas or compressed air generators.

If the turbines driving the boundary layer compressors are near to the engines "hot gas" ducting might be considered.

An alternative would be "cold ducting" from the main compressors to separate combustion chambers of the auxiliary turbines driving the suction compressors.

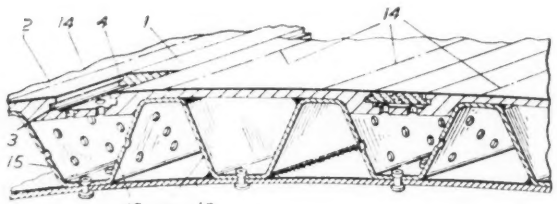


FIGURE 37(*a*). Suction surface with porous strips inserted in the outer skin (British Patent Specification 713426).

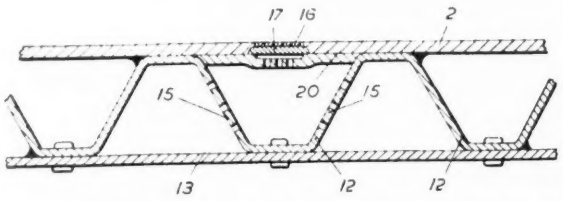


FIGURE 37(*b*). Suction surface with closely spaced perforations in the outer continuous skin (British Patent Specification 713426).

4. Methods of Propulsion for Laminarised Aircraft

Drag and propulsion of laminarised aircraft are much more closely inter-related with each other than on conventional aircraft, and the power plant must be tailored to suit the special requirements in order to reap the full benefit from boundary layer control.

It is not enough to remove part of the boundary layer to control its growth and improve its stability so that it will stay laminar over the whole chord of the wing. It is also necessary to re-energise the sucked air to free stream velocity.

This necessity was soon appreciated by the pioneers of boundary layer control. Pfenninger deduced from his early experiments in Zurich with an aerofoil of 17 per cent. thickness, that if the sucked boundary layer were not accelerated to free stream velocity, *i.e.* with absolute velocity of suction air equalling flight velocity, the effective profile drag coefficient (wake drag plus pump drag) was 0·0038. However, if the sucked air was accelerated to stream velocity, and assuming equal efficiency for suction blower and propeller, the effective drag coefficient was only 0·0023.

In other words, what we call laminarisation through boundary layer suction is not only a method of stabilising a laminar boundary layer, but it comprises implicitly a form of re-energising the boundary layer.

The step from there to boundary layer propulsion is a short and obvious one.

Since the wake drag of a laminarised wing is very small it can be equalised by the momentum of the mass of the sucked air expelled at a velocity somewhat in excess of flight velocity.

Denoting with:—

D_i = induced drag
D_B = body drag plus other form drag
D_W = wake drag of wings and tail surfaces (no suction)
ΔD_W = wake drag of wings and tail surfaces (laminarised aircraft)
T = thrust by conventional methods (propeller, straight jet, ducted fan)
D_p = equivalent pump drag, *i.e.* pump work expressed in form of a drag

the conventional form of propulsion is expressed by

$$T = D_i + D_B + D_W.$$

Applying this method to an aircraft with laminarised wings and tail surfaces

$$T^1 = D_i + D_B + \Delta D_W + D_p.$$

An advantage of Scheme II over Scheme I results if

$$\Delta D_W + D_p < D_W.$$

ΔD_W is very small in case of full chord laminar flow and can be neglected in first approximation. The power plant of a laminarised aircraft must, therefore, supply sufficient power to drive the suction blowers which accelerate the boundary layer to, or beyond, free stream velocity. In addition sufficient thrust must be made available to overcome the induced drag, body drag and any remaining wake drag when the suction blowers are not incorporated in the thrust system.

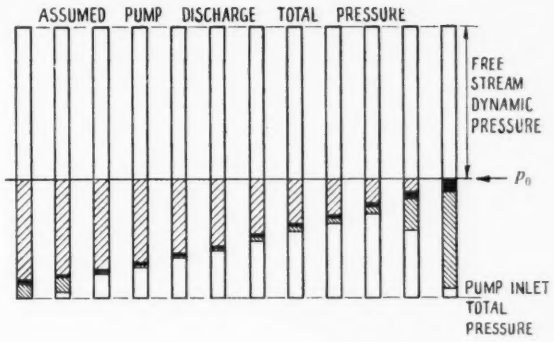

ASSUMED PUMP DISCHARGE TOTAL PRESSURE
FREE STREAM DYNAMIC PRESSURE
p_0
PUMP INLET TOTAL PRESSURE
EXTERNAL LOSS DUE TO SURFACE SUPER-VELOCITY DISTRIBUTION
LOSS THROUGH SURFACE PERFORATIONS
DUCT LOSS (STRIP VALVES FULLY OPEN)

FIGURE 38. Analysis of the various components making up pump power.

Pfenninger investigated various forms of boundary layer propulsion on the basis of cycle studies while engaged on boundary layer research in Switzerland (1943-1948). Part of the studies were later published in Rep. A.M.-88 of the Northrop Aircraft Co.[57]. I am obliged to Dr. Pfenninger for having drawn my attention to this work.

Dr. Head, working on somewhat different lines, has come to conclusions which are generally similar to those predicted by Pfenninger.

Pfenninger's conclusions can be summarised as follows:

(*i*) Propeller-turbines driving suction blowers which eject the sucked air at a velocity somewhat in excess of flight speed have the highest overall efficiency

$$\eta_0 = \frac{\text{thrust} \times \text{flight speed}}{\text{heat input}}$$

defined as the ratio of total thrust work and heat input.

(*ii*) The efficiency η_0 of a gas turbine driving a suction blower (without propeller) approaches that of a propeller turbine as the ratio Pump Drag/Total Drag increases.

(*iii*) A gas turbine driving a suction blower has a higher efficiency η_0 than a ducted fan of the same weight ratio turbine air/fan air, since part of the thrust in the first case is produced by decelerated boundary layer air and not from undisturbed ram air.

(*iv*) The efficiency of a straight jet using suction air is inferior to that of gas turbine cum suction blower.

(*v*) The overall efficiency of a gas turbine cum suction blower can be improved by an additional ducted fan in lieu of a propeller.

While propeller-turbines with power off-take for driving the blower would seem to be a very economical solution (also possessing a very favourable take-off thrust/cruising thrust ratio), it is difficult to use them for driving pusher airscrews, unless they are paired with their axes normal to flight direction and driving co-axial propellers by means of bevels and shafts. With a single turbine driving a pusher propeller it is difficult to get rid of the exhaust gases in the jet which would have to pass through, or over, the hub, or would have to be ducted outside the propeller disc area. Furthermore, the effect on the boundary layer of the inflow into the propeller disc area is not known. There is also the danger of vibrations set up due to the flaps.

An ideal position for pusher airscrews on a laminarised aircraft would be at the tail end. Such an arrangement was once demonstrated by the Douglas Company in the U.S.A., a long shaft drive and gears being used. However, such an arrangement—apart from the mechanical complication—would lead to uncomfortably high disc loadings for multi-engined aircraft.

In view of these difficulties associated with turbine-driven propellers, and also, taking account of their decreasing efficiency at Mach numbers approaching $M = 0{\cdot}8$, the advantage seems to be with ducted fans, the respective turbine energised by ram air driving suction blowers by direct, or pneumatic, coupling.

Besides, a comparison on cycle efficiency only is not sufficient. What matters in the end for a long-range aircraft is not only the weight of the fuel but the weight of the power plant and total fuel.

Design studies on this basis indicate no marked difference between ducted fan engines and propeller-turbines, since on laminarised aircraft, which are very economical in regard to fuel flow per hour, the accent is more on power plant weight than on fuel weight.

There exists no fundamental reason why laminarisation should be confined to wings and tail surfaces. However, laminarising the body will entail very far-reaching consequences in the design of fuselages, for example the elimination of windows, doors and a radical change in cockpit layout. It may entail having the main cockpit far aft with only a small cockpit (perhaps with prone position for the pilot) in the nose of the fuselage, to be used only for take-off and landing. View of the scenery in windowless fuselages could be simulated by camera-obscura effects, or special observation stations could be arranged at the rear end.

However, there is an alternative to laminarising the fuselage surface which might be more practical, although not quite so efficient.

The orthodox method of propelling a body through the air consists in producing a thrust T by accelerating a certain mass m of free-stream air to a higher velocity $v_j > U_\infty$. (Fig. 39).

With a jet the mass m is relatively small and v_j is large. With a propeller m is larger and v_j is smaller, hence for the same thrust kinetic losses $\frac{1}{2}m(v_j^2 - U^2)$ are smaller for the propeller, and the Froude efficiency

$$\eta_F = \frac{2}{v_j/U + 1}$$

is higher than for the jet. Due to the formation and separation of the boundary layer a wake is formed and inside this wake the total head is reduced. This causes the drag D, which has to be equalised by the thrust T.

We have, therefore, the "drag wake," where a negative flow velocity is superimposed upon free stream velocity and the "thrust wake," due to jet or slipstream, where a positive or supervelocity is added to free-stream velocity.

It is obvious that even with infinitely small energy contained in the thrust wake ($\eta_F = 1$), there still remains the loss of energy in the drag wake.

Supposing a system could re-energise the air in the turbulent boundary layer in a plane just ahead of the point where it separates at the rear end of the body, then there would be no drag wake and the horsepower required to perform the re-acceleration would be much less than the shaft horsepower required by a propeller or ducted fan producing a thrust which equals the wake drag.

Obviously, there will be an efficiency of "no return" for such an accelerator mechanism. Preliminary studies have indicated that this limiting efficiency can be lower than that of a propeller.

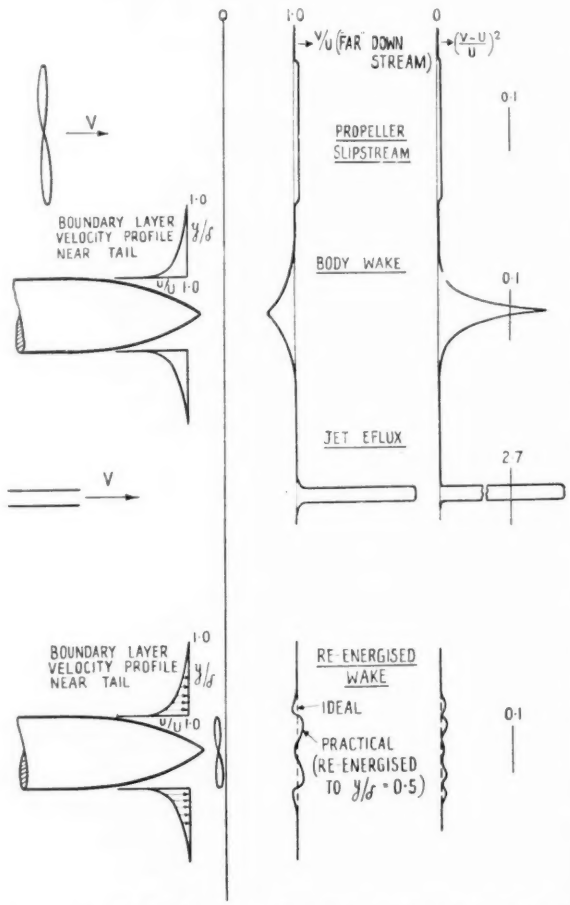


FIGURE 39. Methods of propelling a body through the air and re-energising all the air in the turbulent boundary layer.

Even with an imperfect acceleration which only approximately restores free stream velocity within the boundary layer thickness, substantial improvements are indicated over the conventional method which accepts the body wake as an unavoidable act of providence.

The usual criterion of Froude efficiency hardly applies in this case, when ideally no energy is left in the fluid.

It is more a form of recovering energy which has been lost due to friction and which has not been transformed into heat.

One might liken this form of propulsion to a rocket where the efflux velocity equals flight velocity. In the same way as on a rocket, mass and energy is withdrawn from the body of the rocket ($\frac{1}{2}mv^2$), so is here the virtual mass of the body reduced by the re-acceleration of stagnant boundary layer clinging to the body.

All the foregoing considerations aim at improving fuel economy in cruising flight. However, there is also the need for thrust during take-off and climb, if the laminarised aircraft is to use conventional methods and not, for example, a form of direct lift.

The more cruising drag and thrust are reduced by progressive laminarisation, the more the ratio take-off to cruising thrust reaches uncomfortably large values, and the endeavour to substitute existing power plants by boundary layer propulsion must be tempered by take-off and climb considerations.

It is obviously uneconomical to carry an over-sized power plant over the whole range to the detriment of payload, the function of part of this power plant being restricted to a relatively short duration.

It would seem to be a better policy to have a cruising power plant tailored to suit cruising conditions with best economy supported by a second plant, specifically lighter and possibly of less economical fuel consumption and of shorter life to bolster up the take-off thrust supplied by the cruising plant. The Rolls-Royce "Soar" of exceptionally low specific weight exemplifies this type of take-off power unit.

The amount of thrust required for take-off can be minimised, as shown in Part I, by employing the suction blowers for the dual purpose of laminarisation and the provision of a high lift coefficient. Hence, the two forms of application of boundary layer control are complementary and interdependent on a laminarised aircraft.

Reduction in size of engines corresponding to the greatly reduced cruising thrust of laminarised aircraft (provided that deficiencies in thrust for take-off and climb are compensated by other means) will enable the designer to choose unorthodox engine arrangements. It may lead to the use of gas or air generator plants energising dispersed turbines to drive ducted fans and suction blowers situated in the most suitable positions on the aircraft.

Indirect reduction or elimination of form drag will thus result from the primary reduction of frictional drag, coupled with an increased freedom to place the propulsion unit in the most favourable position and the gas or air producers in the most convenient place.

The recently disclosed experiments with the " Flying Bedstead " indicate the possibility of vertical take-off by means of direct lift from jets deflected vertically downwards.

The additional weight of jets producing a total thrust exceeding the all-up weight of the aircraft would probably be prohibitively large for conventional long-range aircraft where payload is only a small percentage of all-up weight, even if the lift engines are of very low specific weight.

The chances of applying direct lift are better for a highly laminarised aircraft than for a conventional aircraft, owing to greatly reduced fuel weight for the same range and payload. The additional weight of the lifting engines presents itself in highly concentrated form, *i.e.* at high density, which is more convenient than additional low density payload demanding increased stowage volume and wetted surface.

The possibility of replacing, either partially or totally, existing types of jet engines by a form of boundary layer propulsion in cruising flight has another aspect. Noise will be greatly reduced in view of reduced thrust coupled with reduced efflux velocity, a fact that may not only add to the comfort of air passengers, but may also have some significance in the field of military aviation.

This discussion of the demands on the power plant of a laminarised aircraft illustrate the point made at the beginning, namely the need for integration of wings, body and power plant, which is essential to achieve the optimum results from the application of boundary layer control. This integration and the neutralising of drag at the source, so to speak, by suitably infused energy into the boundary layer will demand a very close and intelligent co-operation between aircraft and engine designers.

Exploitation of the finer potentialities of boundary layer propulsion may lead to a splitting up of power units into gas or compressed air generators and dispersed turbines to drive ducted fans.

However, even when neglecting these refined methods for the time being, existing types of conventional by-pass engines, which come nearest to the ideal type of power plant for a laminarised aircraft, are much too large to be suitable for aircraft of greatly reduced thrust requirements.

Taking as an example of a promising application of boundary layer control a four-engined civil air liner or freighter (25,000 lb. payload) to serve the North Atlantic route (London-New York), scaled down units giving roughly 30 per cent. of the thrust of a Conway engine would about fill the bill.

There is nothing in this line on the shelves as far as I know at present.

The alternative, *i.e.* scaling up the aircraft to meet the size of existing units, would lead to excessive sizes and weights.

This situation concerning the power plants for potential laminarised aircraft should be noted by the Powers-that-be. Promotion of boundary layer control for the purpose of laminarisation is of no avail if the development of suitable engines is overlooked. We may

find ourselves unable to harvest the fruits of our labours in the aerodynamic field, because of the lack of engines to power the aircraft of greatly reduced drag.

5. The Effect of Laminarisation on Performance of Aircraft

In view of the effect of surface roughness and sweepback on transition, it would seem to follow that the highest cruising altitude would afford most favourable conditions for a laminarised aircraft due to the reduction with increase of altitude of the all-important parameter U_∞/ν for a given cruising speed.

While this is correct, generally speaking, in so far as difficulties connected with surface finish are concerned, considerations of optimum speed demand an equivalent reduction of induced drag when the frictional drag is lowered.

If reduction of induced drag is attempted by increasing the span of the wings, it will be found by weight analysis that the resulting increase of wing structure weight swallows up a good deal of the benefit that results from drag reduction due to laminarisation.

When comparing performance of a " turbulent " aircraft, *i.e.* an aircraft where the wings have transition at the leading edge and where a turbulent boundary layer exists for the whole wetted surface of the fuselage and tail-unit, with a laminarised aircraft of equal span and weight, it is found that progressive degrees of laminarisation necessitate a corresponding lowering of the cruising altitude for the laminarised aircraft and a simultaneous reduction of cruising thrust and power plant weight to achieve the greatest improvement of range for a given payload, or alternatively, the optimum payload for a given range and fuel weight.

Figure 40 shows the relative effect of various degrees of laminarisation on range, cruising altitude and thrust at optimum cruising r.p.m. All aircraft are of the same all-up weight (on take-off), carry the same payload at the same cruising speed, and are assumed to have the same wing area and aspect ratio.

For the laminarised aircraft suitable allowances have been made for extra weight, *i.e.* weight of suction plant, modifications to wing surface, ducting, and so on.

The basic aircraft, termed " turbulent " aircraft, is a four-engined transport for which detailed weight and performance data were available.

The range of this aircraft at its cruising altitude (100) is marked 100, and it will be noticed that the basic cruising altitude is somewhat above that for which range would be a maximum. In other words, the basic aircraft flies somewhat below optimum cruising speed.

This basic aircraft is now subjected to progressive degrees of laminarisation, assuming either that the same power plant is maintained, or that a progressive reduction of relative thrust and power plant weight takes place as frictional drag is reduced. Cruising speed is maintained constant.

It can be seen that progressive laminarisation without corresponding reduction of power plant results in relatively unimpressive increase of cruising altitude, as compared with that of the basic aircraft, and with apparently no increase of range.

It should be noted, however, that an attempt to increase the cruising altitude of the " turbulent " aircraft by a corresponding amount could only be obtained with additional power plant and with substantial loss of range, assuming payload remains the same.

Figure 41 indicates how substantial increase of range for the laminarised aircraft results if cruising altitude is lowered and power plant weight progressively reduced with increasing degrees of laminarisation.

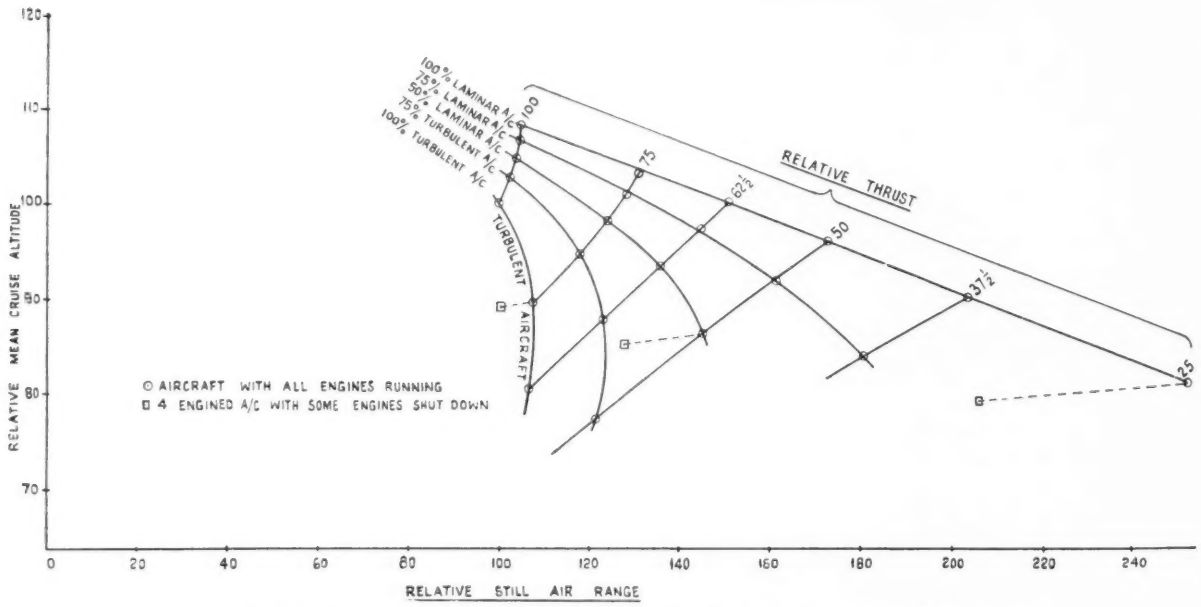


FIGURE 40. Relative effect of various degrees of laminarisation on range, cruising altitude and thrust.

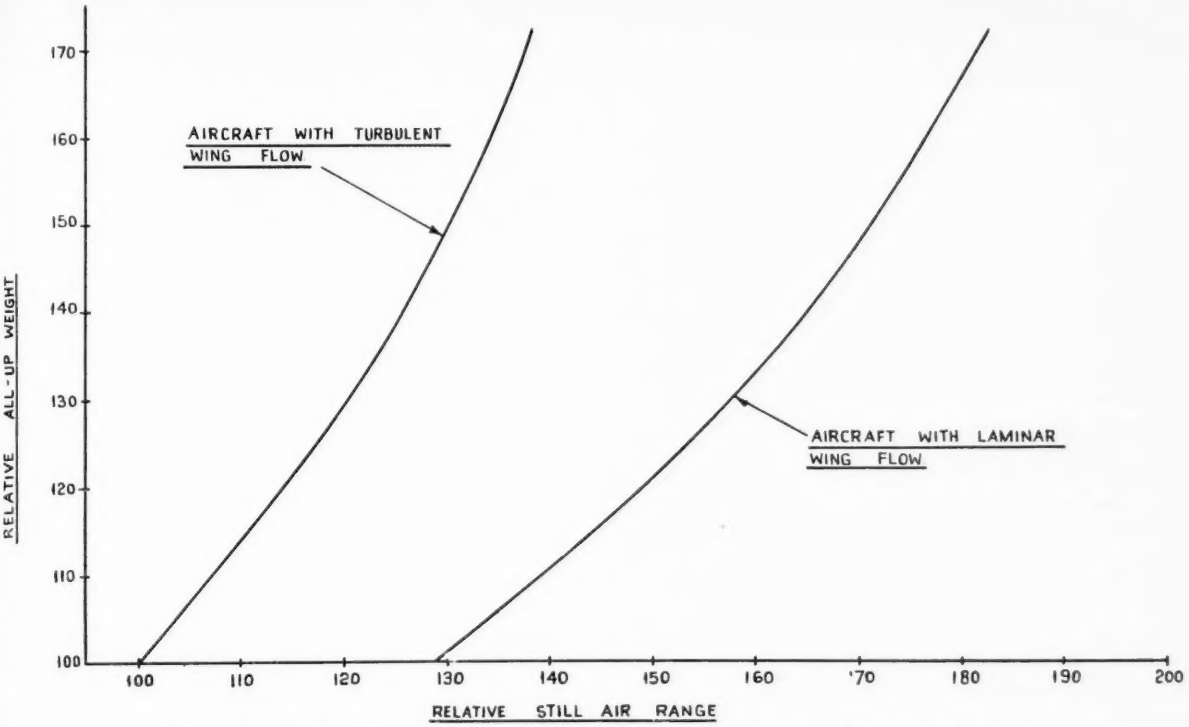


FIGURE 41. Range and weight for two types of aircraft designed to carry the same payload at same cruising speed and taking the same wing loading, aspect ratio and tail volume coefficient.

For example, in the ideal case of an aircraft with complete laminar flow over the whole of its wetted surface, only one-quarter of the thrust of the "turbulent" aircraft will be required, range being about 150 per cent. greater and optimum cruising altitude about 20 per cent. less.

For the more conservative example of, say, 50 per cent. laminarisation, the corresponding figures are: thrust reduction 50 per cent., range increase 45 per cent., and reduction of optimum cruising altitude 14 per cent.

These optimum figures are obtained with power plant weight reduced proportional to thrust reduction.

If the same power plant is carried by the laminarised aircraft as on the " turbulent " one, redundant engines being shut down for cruising flight, the remaining ones operating at optimum cruising r.p.m., the maximum range is reduced, as the dotted lines and square dots in the diagram indicate.

A different approach to that discussed above would consist in designing *ab initio* an aircraft for the basic task of carrying a certain payload at a certain cruising speed over a given range, stipulating at the same time a certain landing speed.

The " snowballing " effect of initial differences of component weights—in this case the reduced weight for power plant and fuel systems for the laminarised aircraft and its greatly reduced fuel weight—is a well-known design phenomenon. Hence, it is not surprising that the aircraft with laminarised wings, carrying the same payload, at the same cruising speed, over the same range, will be substantially lighter and smaller than the equivalent " turbulent " aircraft.

If the range of the basic " turbulent " aircraft should already satisfy requirements—improvements due to lower specific fuel consumption can certainly be expected—the fuel saving due to drag reduction by laminarisation can be converted, of course, into additional payload.

Increase of low-density payload (passengers) necessitates an increase of stowage volume and thus of wetted body surface. A certain amount of stowage volume is gained due to the reduction of fuel and power plant, but the problem of laminarising the surface of the fuselage gains importance for such aircraft designed to carry a large payload of low density.

6. Outlook and Conclusions

Never before in the short history of aviation has the pace of development been so hot and re-orientation been so rapid.

No sooner have we begun to master problems of flight at high subsonic Mach numbers and in the transonic regime, than the still more formidable tasks of supersonic flight loom ahead.

The question " Where do we go from here?" is uppermost in the minds of those who have to plan design and production for the coming decade and beyond.

In the military field the urge towards the attainment

of high supersonic Mach numbers is all-powerful and overshadows all considerations of economy. This urge may lead us, with logical consequence, to the development of the guided missile, not only for defence but also for offence.

The aim for civil aviation is primarily to fly more safely and more economically. The target for military aircraft is to fly faster and higher. These are two very divergent requirements. Hence development of future civil aircraft will, in my opinion, follow separate and independent ways and remain governed by different laws. A repetition of the past where the bomber used to be the pace-maker of the future air liner would seem highly improbable.

There is too much tendency nowadays to transfer the speed fixation of the military into the realms of civil aviation.

Fundamentally there is no improvement of flight economy at supersonic speed; on the contrary, the increase of drag due to the shock waves is barely compensated by the improvements of efficiency of propulsion in comparison with flight at subsonic Mach numbers. The civil operator is attracted by the prospect of better utilisation of aircraft due to reduction of flying time. But the prospect for the aircraft constructor is to enter a rapidly shrinking market and to face ever-increasing and harder competition if this outlook should materialise.

Besides, supersonic flight is only in its infancy and formidable engineering problems have to be solved before flight at high supersonic Mach numbers over long distances will reach the standard of safety and regularity required for passenger traffic.

How does boundary layer control fit into this general picture?

The importance of laminarisation for military applications has obviously receded, at least for such aircraft which are to fly at supersonic speed, except perhaps when extreme range is needed and when strategic considerations permit part of it to be covered at high subsonic Mach number.

Long-range Naval reconnaisance aircraft, submarine spotters and long-range transports should be able to benefit from the improvements in range and fuel economy and from the possibility of designing such aircraft at reduced size and all-up weight for a given range and load.

The importance of reduction in take-off and landing speed and the corresponding reduction of take-off and landing distances, due to the high lift coefficients which are made possible by circulation control, for all deck landing aircraft and many other types of military aircraft, has already been stressed and needs no further emphasis. This applies equally well to civil aircraft, and particular interest exists for the development of aircraft of ultra short landing or take-off distances to compete with the helicopter.

The general prediction is ventured that the bulk of passenger and freight-carrying aircraft will stay subsonic for a long time to come. There may thus be a state of co-existence of the supersonic and the subsonic flight regimes. In the subsonic regime laminarisation through boundary layer control and re-energisation of the boundary layer remain the only fundamental aerodynamic possibilities to improve the economy of flight.

To translate the now well-founded and proven potentialities of boundary layer control, either for lift increase or laminarisation, into engineering reality, emphasis of research must shift from the more academic and theoretical treatment and from wind tunnel testing to flight trials and to the development of practical engineering solutions. Only thus can the necessary "know-how" be gained, this essential link between experiment and practical application.

ACKNOWLEDGMENTS

The author is obliged to the Superintendent of Aerodynamics Division, National Physical Laboratory, for permission to make reference to Dr. John Williams' report and for permission to publish Figs. 3 and 5.

He also wishes to express his thanks to Handley Page Limited for permission to publish this paper and also gratefully acknowledges the assistance he received from Mr. J. B. Edwards (Research Department, Handley Page Limited) in respect to the development of methods and in clearing up several fundamental aspects of some of the problems discussed; also the help of Messrs. Haye and Keates in preparing diagrams.

BIBLIOGRAPHY

1. PRANDTL, L. (1904). Uber Flüssigkeitsbewegung bei sehr kleiner Reibung. *Verhandl. d. III, Intern. Math. Kongr.*, Heidelberg (1904). Also in *Vier Abhandlungen zur Hydrodynamik v. Aerodynamik*, Göttingen, 1927.
2. GRIFFITH, A. A. and MEREDITH, F. W. (1936). The Possible Improvement in Aircraft Performance due to the Use of Boundary Layer Suction. *Unpublished Ministry of Supply Report*, 1936.
3. HOLSTEIN, H. (1940). The Use of Boundary Layer Suction to Maintain the Laminar Characteristics of the Friction Layer, with Special Reference to the Case of Two Dimensional Plates, Exposed to a Parallel Incidence Flow. T.I.B. T 2672 (F.B. 1542), 1940.
4. POISSON-QUINTON, PH. (1948). Theoretical and Experimental Researches on Boundary-Layer Control. *Seventh Int. Congr. Appl. Mech.*, 1948.
   P. REBUFFET and PH. POISSON-QUINTON (1950). Researches on the High-Lift of a Full-Scale Sweptback Wing with Boundary-Layer Control using Air extracted from a Jet-Engine. *La Recherche Aeronautique*, No. 14, p. 39 (1950). Translation N.A.C.A. T.M. 1331.
5. REGENSCHEIT, B. (1941). Eine neue Anwedung der Absaugung zur Steigerung des Auftriebes eines Tragflügels. F.B. 1,474, 1941.
6. WILLIAMS, DR. JOHN (1954). An Analysis of Aerodynamic Data on Blowing over Trailing Edge Flaps for Increasing Lift. Aeronautical Research Council Report No. 17,027, 1954.
7. MILES, F. G. (1939). Sucking Away Boundary Layers. *Flight*, 35, 180, 1939.
8. LYON, H. M. and HILLS, R. (1939). Lift Increase by Boundary-Layer Control. R.A.E. B.A. Dept. Note W.T. 391, May 1939.
   LYON, H. M., BARNES, E. G., and ADAMSON, J. E. (1941). Further Experiments on Boundary-Layer Control as a Means of Increasing Lift. R.A.E. Report B.A. 1669. A.R.C. 5090—Ae. 1788, April 1941.
9. SCHRENK, O. (1928-1940). Tragflügel mit Grenzschichtabsaugung. *Luftfahrtforschung*, 2, 49, 1928. *Zeitschrift für*

Metallkunde, 22, 259, 1931. Versuche mit Absaugeflügeln, *Luftfahrtforschung*, 12, 10, 1935. Grenzschichtabsaugung, *Luftwissen*, 7, 409, 1940. Also N.A.C.A. T.M. No. 974, 1941.

10. BETZ, A. (1939). Beeinflussung der Reibungsschicht und ihre praktische Verwertung. Schriften d. dt. Akad. d. Luftfahrtforschung, Heft 49, 1939.
11. ACKERET, J. (1926). Grenzschichtabsaugung. Z.VDI, 35, 1153, 1926.
12. HAGERDORN, H. and RUDEN, P. (1938). Wind Tunnel Investigations of a Wing with Junkers Slotted Flap and the Effect of Blowing Through the Trailing Edge of the Main Surface over the Flap. Report by the Institut für Aeromechanik und Flugtechnik der Technischen Hochschule Hannover. L.G.L. Bericht, A.64, 1938, pp. 38-56. R.A.E. Library Translation No. 442. A.R.C. 16,882—S. and C. 2919—Perf. 1256. December 1953.
13. SCHWIER, W. (1942). Lift Increase Produced by Blowing a Wing of Profile Thickness of 9 per cent. Equipped with a Slat and a Slotted Flap. M.O.S. GDC/10/1363T, 1942.
14. REGENSCHEIT, B. (1940). Messungen an einem Absangeklappenflügel. N.A.C.A. 23015. Mit und ohne Kwicknase, F.B. 1312, 1940.
15. EHLERS, F. and WALZ, A. (1945). Festschrift zum 60. Gebartstag von Prof. Betz, Göttingen, 1945. Also Experimental Profile Investigations with Boundary-Layer Control. A.V.A. Mono. E. 7. M.O.S. R. and T. 928, 1941.
EHLERS, F. and SCHWIER, W (1940). Blowing Tests on an Aerofoil with Slotted Flap. German F.B. 1274, 1940.
16. WALZ, A. (1940). Absaugeversuche mit kleinen Absaugemengen an einem Flügelprofil 23015 ohne und mit Spreizklappe, F.B. 1159, 1940.
17. HANDLEY-PAGE, SIR FREDERICK (1950). Towards Slower and Safer Flying, Improved Take-Off and Landing, and Cheaper Airports. *Journal of the Royal Aeronautical Society*, December 1950.
18. BUSSMANN, K. and MUNZ, H. (1942). Die Stabilitat der Laminaren Reibungschicht mit Absaugung. *Jahrbuch der Deutschen Luftfahrtforschung*, Part 1, p. 36, 1942.
19. PRETSCH, T. (1942). Umschlagbeginn und Absaugung. *Jahrbuch der Deutschen Luftfahrtforschung*, Part 1, p. 1, 1942.
20. HAHNEMANN, E., FREEMAN, J. C. and FINSTON, M. (1948). Stability of Boundary-Layers and of Flow in the Entrance Section of a Channel. *Journal of the Aeronautical Sciences*, Vol. 15, No. 8, 1948.
21. SCHLICHTING, H. (1942). Die Grenzschicht mit Absaugung und Ausblasen, *Luftfahrtforschung*, 1942.
22. PRESTON, J. H. (1948). The Boundary-Layer Flow Over a Permeable Surface through which Suction is Applied. R. and M. 2244, 1948.
23. IGLISCH, R. (1944). Exakte Berechnung der laminaren Reibungschicht an der langsangestromten ebenen Platte mit homogener Absaugung. *Luftfahrtforschung*, Heft 1, 1944. N.A.C.A. T.M. 1205, 1949.
24. ULRICH, A. (1942). Theoretische Untersuchungen uber die Widerstandsersparnis durch Laminarhaltung mit Absaugung. *Luftfahrtforschung*, 1, 36, 1942.
25. TOLLMIEN, W. (1929). Uber die Entstehung der Turbullenz, Nachrichten d. Ges. d. Wiss. Göttingen. *Math. Phys. Klasse*, p. 21, 1929.
SCHLICHTING, H. (1933, 1935). Zur Entstehung der Turbulenz bei der Plattenströmung, Nachrichten d. Ges. d. Wiss. Göttingen, *Math. Phys. Klasse*, 182, 1933. Amplitudenverteilung und Energiebilanz der kleinen Störungen bei der Plattenströmung, Nachrichten d. Ges. d. Wiss. Göttingen, *Math. Phys. Klasse*, p. 47, 1935.
26. LIN, C. C. (1944-1946). On the Stability of Two Dimensional Parallel Flows. *Quarterly of Applied Mathematics*, Vol. 3, 1944-1946.
27. SCHUBAUER, G. B. and SKRAMSTAD, H. K. (1947). Laminar Boundary-Layer Oscillations and Stability of Laminar Flow. *Journal of the Aeronautical Sciences*, Vol. 14, No. 2, February 1947.
28. LIEPMANN, H. W. (1943). Investigations on Laminar Boundary-Layer Stability and Transition on Curved Boundaries. N.A.C.A. A.C.R. 3H30, August 1943. A.R.C. 7302.
29. SCHLICHTING, H. (1943). The Calculations of the Laminar Friction Layer. F.B. 1480. Also, An Approximate Method for Calculation of the Laminar Boundary-Layer with Suction for Bodies of Arbitrary Shape. N.A.C.A. T.M. 1216, 1943.
30. BURROWS, D. L., BRASLOW, A. L. and TETERVIN, N. (1949). Experimental and Theoretical Studies of Area Suction for the Control of the Laminar Boundary-Layer over a Porous Bronze Airfoil. N.A.C.A. 64A010, N.A.C.A. T.N. 1905, 1949.
31. TRILLING, L. (1950). The Incompressible Boundary-Layer with Pressure Gradient and Suction. *Journal of the Aeronautical Sciences*, Vol. 17, No. 6, June 1950.
32. HEAD, M. R. (1951). The Boundary-Layer with Distributed Suction. A.R.C. 13,897, 1951.
33. RINGLEB, F. O. (1952). Computation of the Laminar Boundary-Layer with Suction. *Journal of the Aeronautical Sciences*, Vol. 19, No. 1, January 1952.
34. WATSON, E. J. (1947). The Asymptotic Theory of Boundary-Layer Flow with Suction; Part I. The Theory of Similar Velocity Distributions; Part II. Flow with Uniform Suction. A.R.C. 10,317, 1947.
35. THWAITES, B. (1949). The Development of the Laminar Boundary-Layer under Conditions of Continuous Suction; Part II. Approximate Methods of Solution. A.R.C. 12,699, 1949.
36. THWAITES, B. (1946). An Exact Solution of the Boundary-Layer Equations under Particular Conditions of Porous Suction. A.R.C. 9,656 (R. & M. 2,241), 1946.
37, 38. SCHLICHTING, H. and BUSSMANN, K. (1943). Exact Solutions of the Boundary-Layer with Suction and Blowing. *D.A.L.* 7B, 1943.
39. MANGLER, W. (1948). A.V.A. Monograph, Reports and Translations, No. 1,001 (Ref. GDC/3242T), 1948.
40. PERRING, G. W. A. and DIPROSE, K. V. (1936). Note on the Effect of Suction on the Stability of the Laminar Boundary-Layer. R.A.E. Report No. B.A. 1,395, 1936.
41. KAY, J. M. (1948). Experimental Examination of Boundary-Layer Flow Along a Flat Plate with Uniform Suction. A.R.C. 11,476, 1948.
42. HEAD, M. R. and JONES, SIR MELVILL (1951). The Reduction of Drag by Distributed Suction. *Third Anglo-American Conference*, Brighton, 1951. Royal Aeronautical Society, London.
43. ACKERET, J., RAS, M. and PFENNINGER, W. (1941). Verhinderung des Turbulenwerdens einer Grenzschicht durch Absaugung. *Die Naturwissenschaften*, 622, 1941.
44. PFENNINGER, W. (1946). Untersuchungen über Reibungsverminderung an Tragflügeln, insbesondere mit Hilfe von Grenzschicht-Absaugung. Mitteilungen aus dem Institut fur aerodynamik. E.T.H., Zurich, Heft 13, 1946.
PFENNINGER, W. (1949). Laminar Flow Airfoils with Boundary-Layer Suction. *Interavia*, March 1949.
PFENNINGER, W. (1949). Experiments on a Laminar Suction Airfoil of 17 per cent. Thickness. *Journal of the Aeronautical Sciences*, April 1949.
45. BURROWS, D. L. and SCHWARTZBERG, M. S. (1952). Experimental Investigations of an N.A.C.A. 64A010 Airfoil Section with 41 Suction Slots on Each Surface for Control of Laminar Boundary-Layer. N.A.C.A. T.N. 2,644, 1952.
46. BRASLOW, A. L., VISCONTI, F. and BURROWS, D. L. (1948). Preliminary Wind-Tunnel Investigations of the Effect of Area Suction on the Laminar Boundary-Layer over an N.A.C.A. 64A010 Airfoil. N.A.C.A. R.M.L. 7,415, 1948.
47. BRASLOW, A. L. and VISCONTI, F. (1950). Further Experimental Studies of Area Suction for the Laminar Boundary-Layer on a Porous Bronze N.A.C.A. 64A010 Airfoil. N.A.C.A. T.N. 2,112, 1950.

48. RASPET, DR. AUGUST (1951). Mechanism of Automatic Trailing Edge Suction. Naval Low-Speed-Flight Research Program, Mississippi State College Research Report 1, 1951.
49. FAGE, A. (1943). The Smallest Size of a Spanwise-Surface Corrugation which Affects Boundary-Layer on an Aerofoil. R. & M. 2,120, 1943.
50. GREGORY, N. and WALKER, S. (1950). The Effect on Transition of Isolated Surface Excrescences in the Boundary-Layer. A.R.C. 13,436, 1950.
51. LOFTIN, L. K. (1946). Effects of Specific Types of Surface Roughness on Boundary-Layer Transition. N.A.C.A. A.C.R. L5J29a, February 1946. N.A.C.A./T.I.B./1,272.
52. KLANFER, L. and OWEN, P. R. (1953). The Effect of Isolated Roughness on Boundary-Layer Transition. T.M. Aero. 355, March 1953.
53. SCHILLER, L. (1932). Handbuch der Experimental Physik, Part 4, Leipzig, pp. 189-192, 1932.
54. DRYDEN, H. L. (1953). Review of Published Data on the Effect of Roughness on Transition from Laminar to Turbulent Flow. Paper presented at the Aerodynamics Session, Twenty-first Annual Meeting, I.A.S., New York, January 1953. *Journal of the Aeronautical Sciences*, Vol. 20, No. 7, pp. 477-482, July 1953.
55. TANI, I., HAMA, R. and MITUISI, S. (1940). On the Permissible Roughness in the Laminar Boundary-Layer. *Aeronautical Research Institute, Tokyo*, 199, Tokyo, 1940.
56. LACHMANN, G. V. (1954). Laminarisation through Boundary Layer Control. *Aeronautical Engineering Review*, Vol. 13, No. 8, p. 37, August 1954.
57. PFENNINGER, W. (1949). Report A.M.-88 of the Northrop Aircraft Co., 1949.

DISCUSSION

R. A. SHAW (Ministry of Supply, *Associate Fellow*): It was about a generation ago that Sir Bennett Melvill Jones lectured to the Society on the streamlined aeroplane. They could look back and see the effects of his thesis on aircraft today. Since his lecture they had eradicated a lot of things that stuck out of aeroplanes and reached the stage that the present generation of high performance aircraft had a profile drag which was perhaps only 50-70 per cent. more than the turbulent skin friction of the basic surface.

It was significant how content they had been with this situation that the makers of swept-wing aircraft had for ten years been unaware that by the nature of their plan form they were bound, generally speaking, to have turbulent flows over their wing surfaces. They were, obviously, content to let it be so. Now Dr. Lachmann's lecture marked a new stage, and they had to think of building aeroplanes still more intelligently and to try to get that extra advantage which could be got by controlling the boundary layer itself.

Dr. Lachmann's lecture would take many people plenty of time to digest. Obviously he had given profound thought to the subject. If they were to apply those aids and make a success of them, many other people must put in the same kind of effort to their application in engineering.

Some of them would be struggling with blowing over flaps, a form of boundary layer control, to try to produce higher lift for landing aeroplanes. Very few he thought would have been involved yet in laminarising aeroplanes and not many would have had much contact with boundary layer control and he would like to convey to them his own feeling of flying in an aeroplane subject to boundary layer control.

In the United States he had the privilege of visiting Professor August Raspet at Mississippi State College and flew in two of his gliders. The first of these had the upper surface of the wing perforated over the whole span and was used for a study of high lift by porous suction. He had flown in the second seat of this glider and had under his control the switches of a couple of blowers each about the size of a hair dryer, one for each wing, driven by an accumulator. The wing was fully tufted. They were released at 5,000 ft. and, by arrangement, the pilot deliberately stalled the glider. He had watched while the wing flow broke down and all the tufts went wild. It was very quiet. He had reached over and switched on the blowers. In the second or two that the blowers took to run up to speed the tufts straightened out and the wing became suddenly and smoothly buoyant. It was easy to believe, as he was afterwards told, that the stalling lift coefficient had increased 50 per cent. In another run, repeating the experiment, he tried the effect of switching on one blower only. The effects were even more impressive!

He then flew in the second glider which had a part of the wing span, close to the wing root, specially prepared for laminar flow studies with porous suction. The same kind of hair dryer blowers again provided all the suction power. He was given a stethoscope with a long pitot tube which he could hold out over the side of the cockpit to listen to the flow.

Although he was anxious to find the difference between putting the blowers on and off, he could not notice it, until he discovered that he had not connected the rubber tube. When he did so it was most impressive to find how the clatter of the turbulent flow could be hushed to the smooth hiss of laminar flow, at will, simply by turning on the suction.

It was a big step from that little glider experiment to the aeroplane that Dr. Lachmann envisaged, but he believed that that step was possible. They had had to wait 20 years or more for the fulfilment of Sir Bennett Melvill Jones' prophecies. They might have to wait another 20 years for the fulfilment of Dr. Lachmann's prophecies, but they had the tools in their hands. With the jet engine they had the means to handle large quantities of air and submit the air to whatever pressures they chose. They had that possibility now which would produce results in the next generation.

I. M. DAVIDSON (National Gas Turbine Establishment, *Associate Fellow*): Towards the end of his paper, Dr. Lachmann showed what a large breach they might

expect to find between the military and the civil fields in years to come. With this in mind, he thought it would be difficult to become excited about high lift for the military machine, so that one should think more of what could be done with it to improve the overall economy of operation. Dr. Lachmann pointed the way towards the end of Part I of the paper, where he mentioned a machine with a take-off and landing distance of 500 ft. That, perhaps, was a little optimistic for a balanced field length, but something of that order would certainly seem to be possible if they were to believe the figures in the paper showing the lift coefficient produced by blowing over the flaps.

In that connection, he would like to ask two questions. In civil operation now, they had to use a C_L margin of quite appreciable magnitude to take account of gusts and other such contingencies. If they were to apply any high lift system to a civil machine, they really must go to the limit, because they would not be trying to compete with the normal fixed wing aircraft. If they did this, they would be trying to replace the large helicopter.

In one part of the paper Dr. Lachmann mentioned the extremely good handling characteristics of the modified Grumman Panther. Did he feel that the intelligent application of boundary layer control for high lift would not only bring about high lift itself, but also such handling characteristics and freedom from stall as would enable them to whittle down these margins and, thereby, gain a worthwhile bonus?

Again in Part I of the paper Dr. Lachmann showed a series of curves in which he plotted lift coefficients, albeit theoretically, going up to values of about 10 and over a range of aspect ratios and a range of thrust to weight ratios. These thrust to weight ratios went from the very low values which one would have with a laminarised machine, right up to the values which one would have to expect to use for direct jet lift.

When plotting this information in a somewhat different form, however, one found that these very short take-off and landing runs really began to pay off when a thrust to weight ratio of about 0·4 to 0·45 was achieved. This was, of course, much more than one would have at take-off from the cruising engines of a machine of this sort. Did Dr. Lachmann feel that it was important that they should consider the use of special boosting engines, even for short range civil operations?

The necessity for shortening his talk did not allow Dr. Lachmann to deal verbally with one important point about the laminarised machine, but it was referred to in the paper: that was, his over-riding thesis of an improved integration of the power plant and the airframe. One point which had not been brought out fully in this connection was that for some years the engine specific weight had been steadily reducing. Could it be that on the airframe side of the business it was not generally realised that one of the reasons why this weight had come down was that the engine itself was becoming much more closely integrated and much more compact? If they wished to break it up in order to use, perhaps, centralised gas generators feeding turbines in other parts of the aircraft, there was no fundamental reason why they should not but they would have to face a large increase in the engine weight. He did not think that this factor had so far been taken fully into account.

The concept of super-circulation was mentioned in connection with the high lift work. If he understood correctly, the interpretation of this extra lift which the term super-circulation implied was stated to have arisen in America. He was not altogether happy about this, because from what he could find in the literature it seemed that the idea itself was quite fully discussed in Germany in 1938 by Hagedorn and Ruden, although they did not give the phenomenon its name. Again in 1947 Poisson-Quinton and in 1948 Metral and Zerner among others used, in France, the term *hypersustenation* which they applied, not to the whole of the lift increment, but specifically to that part which remained when allowance was made for the direct component of the jet reaction. As this super-circulation phenomenon was one which might well become more important in the future, he thought that he should draw attention to these historical facts.

W. H. PAINE (Westland Aircraft, *Associate Fellow*): For a long time they had been arguing among themselves over the advantages of boundary layer control, and in particular with regard to increase in lift.

He agreed very much with Dr. Lachmann when he stated that the emphasis of research must now shift from the more academic treatment to the solution of the engineering problems. They were lagging quite a long way behind on this. A lot of work had been going on, particularly in America, and it was his opinion that the design studies, tenders, etc., on which various aircraft firms were working would be considerably enhanced if they knew rather more about this particular kind of application.

In his paper, Dr. Lachmann spoke of being able to do away with the aileron and flap by the use of some form of circulation control. If they could develop circulation control, he felt that they ought to be able to extend it not only to ailerons and flaps, but to all the other controls, especially with very short fuselage type aircraft.

D. JOHNSON (Royal Aircraft Establishment): At the R.A.E. they had attempted in flight to control the boundary layer on a sleeve fitted to the upper surface of the wing of jet aircraft. This work was really a continuation of the work done at Cambridge, and was in a way further connected because Dr. Head was also responsible for initiating the programme at R.A.E. and considerable success was achieved before he returned to Cambridge.

The sleeve was of about 8 ft. chord and 4 ft. 6 in. in width. The surface was porous from something like 6 per cent. chord back to the trailing edge and was designed to operate at a lift coefficient of about 0·2 and a Reynolds number of approximately 20 million. He was glad to say that they achieved full chord laminar flow which was rather encouraging. They were also able to maintain laminar flow up to something very near the

critical Mach number of the section. He could not give any details of the construction of the sleeve, but they probably realised that with full chord laminar flow the drag reductions to be expected were very impressive. Taking into account pump drag, they obtained a reduction in drag (excluding induced drag) of better than 70 per cent.

Dr. Lachmann had mentioned some of the difficulties associated with laminar flow winds and he agreed. Surface roughness, of course, was a serious problem on all laminar flow surfaces, as they all knew. They were worried with it to some extent, and they might have been better off had they been able to use a highly polished surface. By "roughness" he referred to grit and dirt, as distinct from the surface texture.

Another matter that was always raised in connection with laminar flow wings was the question of roughness caused by insect contamination. Flies impinged on the leading edge of an aircraft wing. This contamination was nearly always a problem, and unfortunately had to be cleaned off. Once insects had become dried on, it was not easy to remove them. However, they affected only the first few per cent. of the wing surface, and in their experiments they protected the wing during take-off, with a simple cover over the first few per cent. of the chord. It might be that something in this form could be developed for use on larger spans.

He thought that the standard of construction that it was possible to achieve in production approached the standard that they required in the case of laminar flow wings. However, skin joints might be troublesome in that they gave discontinuities in the shape of the profile locally around the joint and might prove somewhat difficult.

J. W. FOZARD (*Graduate*): He agreed wholeheartedly with Dr. Lachmann on the question of lift augmentation by blowing in that such devices did not provide a unique solution to the problems of take-off and landing.

One had to be reasonable in these matters and, as he saw it, the problem was essentially one of designing for a reasonable take-off and landing performance without blow and then to make it very good by the application of blow. In this way one avoided the very large values of induced drag due to very high C_L increments. In trying to achieve very large C_L increments the induced drag became so large that the aircraft was flying on the unstable part of the drag curve. This was particularly noticeable at take-off when the C_L increment due to blow was a much larger percentage of the basic unblown lift coefficient than at landing.

However, the induced drag problem did arise during approach and landing. As Dr. Lachmann pointed out, in this condition they required a large increase in the "flat plate" drag D_0. Using the engine as the source of blow, they knew that for the larger C_L increments they required high pressures in the ducts and this meant that the engine must be run at high r.p.m. This in turn implied a large thrust which became embarrassing on the approach to landing. A thrust reverser would be of especial benefit under these conditions—enabling high engine r.p.m. to be used (giving high duct pressures) while at the same time increasing the "flat plate" drag D_0 to enable a steep approach to be used.

He wondered, in view of the importance of induced drag, whether there were any methods of reducing drag by some form of B.L.C. One approach was blowing air out of the wing tips to obtain an increase of aspect ratio. One must remember that the induced drag was proportional to the square of the span loading and so any artificial increase of span would have an important effect on D_i. He was an engineer, not a scientist but he felt that some fundamental method of reducing the induced drag eventually would be achieved. If they were to make full use of the potentialities offered by lift augmentation by blowing, some mitigation of undesirable features such as large induced drag was essential.

N. E. ROWE (Blackburn and General Aircraft Ltd., *Fellow*): The Society was privileged to have this paper from Dr. Lachmann, because it set a new stage in progress in aerodynamics, which, as Dr. Lachmann recalled in his historical note, was started by Prandtl and was most certainly followed by Sir Bennett Melvill Jones in his classic papers from time to time. The paper would have a great influence on thought and action.

The paper referred to a new integration between power plant and airframe, and a number of others had referred to this. Thought had been moving in that direction for some time, but what the final outcome of that integration would be it was difficult now to foresee.

An earlier speaker expressed the view that the use of blowing, with gas from a central gas generator and having means of using the gas in various parts of the aircraft, might lead to excessive weights. He thought that the integration would come more subtly, and in such a way that a great increase in weight would be avoided. As Dr. Lachmann said, one of the likely difficulties was that they would not have the engines available to allow them to take advantage of the great aerodynamic advances that Dr. Lachmann foresaw.

That, he thought, would be another major reason why civil aviation would tend to depart in its progress from military aviation, because civil aviation wanted increased safety and economy. Economy was absolutely the keynote. It was the challenge of the day. If they had greater economy, they would have greater use of the air for transport, and he was sure this must come. But if it were obtained by these methods of boundary layer control, particularly laminarisation—a new word which he did not like very much—he thought they would need smaller engines. They would need different sorts of integrations of the engine and the airframe, and he thought they would be quite different from those in military aeroplanes now and those one would expect during the next generation. It was an extremely important point and one that should be recognised and grasped as quickly as possible.

The trend of events in the past had always been that major advances were necessarily coupled to the military research and development. That was probably more true of engines than of airframes. Civil aviation had used engines that had been developed for military use, and in the past this had proved to be an extremely

advantageous arrangement. With certain engines, notably the Dart, it had not been so, but now they wanted a completely new range, because they needed them to be smaller. They needed them split into two parts, one for cruising and one for take-off. That seemed to him to be a major departure which went with this new form of aerodynamics, and he would like the author's opinion on it.

Dr. Lachmann said in his paper that he did not see the purpose of civil aviation flying supersonic, because at a certain stage the drag rise was so rapid that there was a rather violent diseconomy. That was probably correct, but he did not think it would necessarily limit civil aviation to a subsonic regime. Civil aviation sold its schedule—it sold time—and if there were a competitive selling of time, he thought that airlines would go to the limit in these things.

Dr. Lachmann had emphasised, he was sure quite rightly, the need now to concentrate on the plumbing. He thought that the simple approach—and this might be a first stage—was to use the simple form of plumbing required for air under the modest pressures needed for blowing. This was what had been done in the United States. When one used pressure, the ducts and methods of application were immensely simplified; it was a completely positive system and valuable improvements could be obtained even by application to existing aircraft.

He would hazard a guess, therefore, that the opening approaches to boundary layer control generally would be by this pressure method. It was open to their hands from the turbine, and it seemed to him to be something that could be done simply in aeroplanes without undue fuss and without a great deal of weight, and which could be applied to existing aircraft. He would like Dr. Lachmann's views on that also.

A remark was made by an earlier speaker that he thought the best method was to design aeroplanes without any of these things—perhaps he was referring only to pressure—and then to use them to improve the aeroplane. That might be sound as an opening gambit when feeling one's way, but he felt certain that, as with other great improvements, to get the most out of it one must build the improvement into the design. If not, the "snowball" to which Dr. Lachmann referred simply operated against one.

The potential gains were great. Improved economy resulted from using a smaller, lighter, less powerful aircraft for a given transport task. That was what could be obtained by tackling design *de novo* on the lines that Dr. Lachmann had suggested.

THE PRESIDENT: It had been a most stimulating paper, and one which might well mark a point of change in design philosophy, but through it there loomed large the importance of practical engineering and, to him personally, the difficulty of making wings which must be thinner and thinner and yet take more and more fuel, as well as some form of boundary layer control.

No speaker had mentioned the consequences of power failure—that was, of course, important, as they could not depend entirely on power unless the consequences of failure were not serious.

Another point which occurred to one when listening to a lecture such as this was: Why had not they done more on these lines? There were, of course, plenty of reasons in answer to this, but nevertheless it was rather surprising that progress had been so slow. They must, however, all agree that they would be forced to give much more attention to the question of boundary layer control in the future.

DR. LACHMANN'S REPLY

MR. DAVIDSON: In regard to the C_L margin due to the stipulated ratio of stalling speed to approach speed, he agreed wholeheartedly that with modern methods of super-circulation it should become possible either to reduce or to eliminate this margin altogether.

Ways and means existed for doing that, and he also knew that Mr. Davidson had some thoughts of his own on this subject but for obvious reasons, he could not discuss the subject any further.

According to a talk which he had with Mr. Attinello, lateral control at low speed with the blow-out flap was greatly improved compared with the ordinary aircraft, and the success of the demonstrations of the Attinello flap on the Grumman "Panther" was not only due to the extra twenty knots reduction in minimum speed, but also due to the greater confidence of the pilots in flying at the reduced speed which resulted from the lift increase. This improved controllability and the general feeling of confidence enhanced the favourable impression created by the demonstration of the blow-out flap.

During take-off and climb to cruising altitude a laminarised aircraft was, of course, operating on equal terms with a conventional one, while thrust requirements during the cruise were greatly reduced.

In order not to penalise the laminarised aircraft by the weight of an over-sized power plant giving excessive thrust during the cruise, it was advantageous to split the power plant into cruising engines tailored to the requirements of low thrust, and additional auxiliary engines of very low specific weight for take-off and climb only. Even with this method it would be uneconomical to try to achieve thrust to weight ratios (at take-off) of the order mentioned by Mr. Davidson. One might therefore conclude that extremely high lift coefficients were not economical for laminarised aircraft. However, the use of these special boosting engines might be, with advantage, considered on conventional aircraft for short range operations when the emphasis was on exceptionally short take-offs. The effect of putting on more span or more thrust was the same, but putting on more thrust by means of boost engines might be a simpler and lighter method, provided that the boost engines had a very low specific weight.

He appreciated Mr. Davidson's argument that the weight of the power plant might be increased by the

method of using centralised gas generators and dispersed turbines, and this disadvantage might over-ride the attractive possibilities which this system offered in other respects.

He was obliged to Mr. Davidson for his remarks about "super-circulation." He was aware that the conception did not originate in the U.S.A., but he had been under the impression that the term "super-circulation" was more popular in the United States than in Great Britain.

MR. PAINE: On laminarised aircraft it was essential that the flow be kept laminar right back, or almost back, to the trailing edge, and it paid a dividend to avoid conventional control surfaces. Some form of circulation control or blow-out spoilers could replace the ailerons and could also be applied to the tail surfaces, although all movable horizontal and vertical tail surfaces might be a simpler proposition.

MR. JOHNSON: He was very grateful to Mr. Johnson for having obtained clearance to make this very important and first public announcement that R.A.E. in their flight studies had been able to get laminar flow over the whole wing up to the critical Mach number. Had he known that this statement would be made, he would have changed the introduction to his lecture and given it the prominence which this achievement deserved.

MR. FOZARD: He was not aware that the idea of blowing out in spanwise direction had been tried. He remembered that the Americans once reported that, when they tried suction on the Cessna, they got an apparent reduction of induced drag, but they did not know quite how and why. The opinion was expressed that by blowing, the end vortices shifted a little farther outward. Whether that was true, he did not known. It might be interesting to try blowing out in a spanwise direction, but he feared that very large quantities would be required to achieve any effect and it might be lighter to put on a little more span.

MR. ROWE: The engine question was extremely important. They were now going ahead very nicely on the aerodynamic side and were beginning to gain engineering "know how." but if they wanted to get a satisfactory boundary layer aircraft—he was talking about a laminarised aircraft—the right engines must be available. He had drawn attention to this point in the paper, but he believed that certain engines which were available could be modified to suit the purpose, or could be scaled down.

The possibility of "selling time" depended very much on the route. In other words, it seemed only of value to increase the speed if the utilisation of the aircraft could thus be improved. Taking, for example, the North Atlantic crossing, the utilisation of the aircraft per annum would hardly be affected by variations of speed of the order of 50 to 100 m.p.h. since this would not increase the number of crossings possible within twenty-four hours.

In assessing realistically the advantage of arriving half an hour or even an hour earlier on the other side of the Atlantic, one should compare the reduction in flying time gained at considerable technical effort with the unavoidable secondary delays on the ground. Air travellers generally had to assemble about one and a half hours prior to departure at the air terminal or half an hour prior to departure at the aerodrome. There were often considerable delays, sometimes hours, due to bad weather and other reasons. After landing on the other side of the Atlantic passengers spent generally about two hours in order to clear immigration and customs and then another three-quarters of an hour in travelling to their hotel. The technical efforts to shorten actual flying time across the Atlantic should bear some relation to those secondary delays.

In crossing the Atlantic at supersonic speed the actual flying time became of the same order as the secondary delays.

He would also like to mention that he belonged to an organisation which had to earn its bread and butter by making aircraft, and their point of view was to sell more and better aircraft instead of fewer and faster machines.

As far as boundary layer for lift increase was concerned, the simple blow-out scheme was obviously the lightest, but unless the engines were designed for it, it might not always be the best and most economical one, because a lot of mass flow was required if large lift increases were desired, and on landing, with throttled engines, it might not be possible to get enough mass flow out of the engines. Therefore he thought it was not out of the question that a more economical method, practically as simple as the blow-out system, would be developed based on sucking the boundary layer away.

The Conjugate Load Method in Structural Analysis

by

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1. Introduction

The problem of analysing the behaviour of a structure as a combination of members joined together, may be stated in the form of a question—What system of loads must be applied to a single member of the combination so that its behaviour as an isolated member is identical to its behaviour as an integral part of the structure? The question becomes clearer upon defining some of the terms used in stating the question.

Loads—The forces and couples necessary for the equilibrium of the isolated member. They are related to the actual external loads applied to the structure, hence are termed "conjugate."

Member—Line or Axial—A part of the structure measured along a line from one joint or support of the structure to another joint or support, without a break (gap or kink) in the continuity of the line. Line-members may be classified into strings, bands and bars, with cross-sectional dimensions relatively small compared to length along the axis.

Surface or shell—A part of the structure measured along two intersecting curves which define a reference surface, without a break in the continuity of the surface, from one joint to another. Thickness dimensions normal to the reference surface, are relatively small compared to lengths in the surface.

Joint—The region of intersection of two or more members. Supports are included in the definition of joint.

Behaviour—For convenience in analysis, behaviour in general is divided into three partial behaviours:—

(*i*) *The mechanical behaviour*—the relationships which exist between the external loads (loads include reactions) and the internal forces. An equilibrium behaviour is expressed by the equilibrium equations.

(*ii*) *The geometrical behaviour*—the set of relations which describe the change in configuration that occurs during the application of load.

(*iii*) *The resistive behaviour*—the relationship which is assumed to exist between the stress-tensor and the strain-tensor at a point in the material of the member. Resistive behaviours depend primarily on the nature of the material hence such designations as rigid, elastic, plastic, viscous, and so on.

The answer to the question stated, is obtained by identifying the geometrical and resistive behaviours of the isolated members at the joints of the structure. Identification leads to a finite number of equations in the case of a structure with a finite number of joints and to the differential equations for the conjugate loads for a structure with an infinite number of joints as in the case of a surface-member.

Notation

$\wedge$	indicates a vector product
.	indicates a scalar product
Q	moment of area under moment diagram for transverse loads (P) only, with the span L, simply supported. (Subscript here denotes moment centre.)
δ	displacement of end of member at right angles to the axis of the member
e	displacement of end of member along the axis
θ	angle of rotation in bending
ω	angle of rotation in torsion
a	cross-sectional area of member
u, v, w	displacements in directions x, y, z, respectively
I	moment of inertia of cross section with respect to the neutral axis of bending
V'	shear in the conjugate beam
M'	moment in the conjugate beam
X, Y, P	loads
R	reactions
$\mathbf{F}$	force per unit of length
$\mathbf{M}$	moment per unit of length
$\mathbf{p}$	force as load per unit of area
$\mathbf{c}$	couple as load per unit of area
$\mathbf{t}$	unit-vector tangent to a curve
G_{12}	geodetic curvature of curve C_1, in the surface Σ_{12}
G_{21}	geodetic curvature of curve C_2, in the surface Σ_{12}
$\mathbf{r}$	shape-vector which locates element of area in section
s	length along a curve C
ds	differential length along C
σ	stress (per unit area)
ε	strain in middle surface (per unit length)

Originally received June 1954.

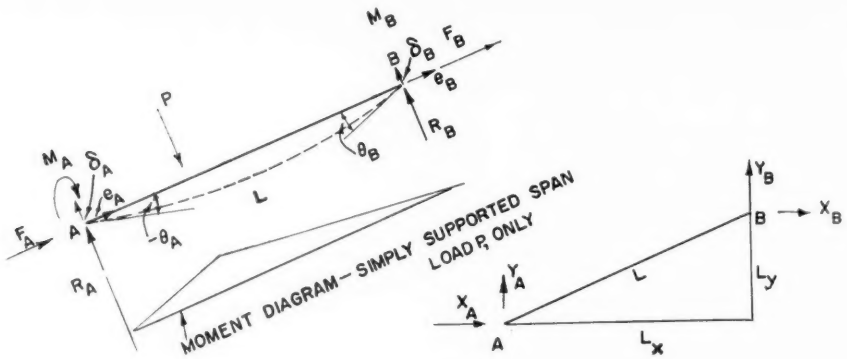

FIGURE 1. Single member in extension and bending.

$\bar{e}$ strain-tensor
$\bar{\sigma}$ stress-tensor
E modulus of elasticity in tension and compression
μ Poisson's ratio
e_1 first invariant of the strain-tensor, $\bar{e}$
$\bar{I}$ unit dyad
α angle between unit tangent vectors $\mathbf{t}$
Σ middle surface
da differential area
ΔN change in normal curvature
ΔT change in geodetic torsion
h thickness of plate

$$G=\frac{E}{2(1+\mu)}, \quad \bar{\mu}=\frac{\mu}{1+\mu}, \quad \varepsilon_0=\bar{\mu}(\varepsilon_1+\varepsilon_2)$$

$$\lambda=\frac{\mu E}{(1+\mu)(1-2\mu)}, \quad D=\frac{Eh}{1-\mu^2}, \quad K=\frac{Eh^3}{12(1-\mu^2)}$$

$$\bar{A}=-(\Delta N_{12}+\Delta N_{21})$$

2. Behaviour of a Bar—Extension and Bending

It is assumed in this analysis that the resistive behaviour of the material is such that the geometrical behaviour is limited to small displacements and rotations. As a consequence the equilibrium equations for the member may be expressed for its undeformed shape. A further consequence is that the geometrical behaviour may be expressed in the usual elementary form. For a straight bar, loaded as shown in Fig. 1, the elementary behaviour, extension and bending, is given by the following equations:

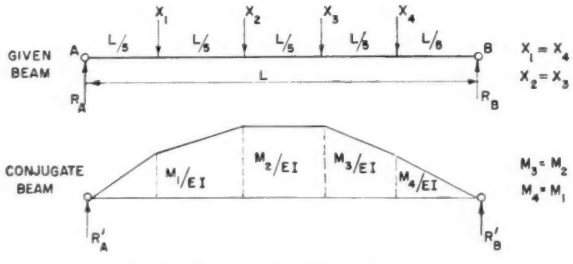

FIGURE 2. Simply supported beam.

$$\left.\begin{aligned} X_A L^2 &= L_x L F_A - L_y (M_B - M_A) \\ Y_A L^2 &= L_y L F_A + L_x (M_B - M_A) \\ X_B L^2 &= L_x L F_B - L_y (M_A - M_B) \\ Y_B L^2 &= L_y L F_B + L_x (M_A - M_B) \end{aligned}\right\} \quad . \quad (1)$$

$$L^2 F_A = -L^2 F_B = ELa(e_B - e_A) = Ea[L_x(u_B - u_A) + L_y(v_B - v_A)] \quad . \quad (2)$$

$$\left.\begin{aligned} M_A L^2 &= -2(2Q_B - Q_A) - 2EIL(2\theta_A + \theta_B) - 6EI(\delta_A - \delta_B) \\ M_B L^2 &= 2(Q_B - 2Q_A) + 2EIL(\theta_A + 2\theta_B) + 6EI(\delta_A - \delta_B) \end{aligned}\right\} \quad . \quad (3)$$

Rotations and deflections at any section of a horizontal simple beam loaded as shown in Fig. 2, are obtained as shears and moments in the corresponding conjugate beam[1].

$$\left.\begin{aligned} -EI\theta_1 &= EIV_1' = \frac{M_1 L}{10} + \frac{M_2 L}{5} \\ -EI\theta_2 &= EIV_2' = \frac{M_2 L}{10} \end{aligned}\right\} \quad . \quad (4)$$

$$\left.\begin{aligned} -EIv_1 &= EIM_1' = \frac{L^2}{25}\left(\frac{5}{6}M_1 + M_2\right) \\ -EIv_2 &= EIM_2' = \frac{L^2}{25}\left(M_1 + \frac{11}{6}M_2\right) \\ M_1 &= \frac{L}{5}(X_1 + X_2) \\ M_2 &= \frac{L}{5}(X_1 + 2X_2) \end{aligned}\right\} \quad . \quad (5)$$

Torsion—For a straight bar loaded by concentrated torsional couples (Fig. 3) the equilibrium equation is

$$T_1 + T_2 + T_3 + T_4 = 0.$$

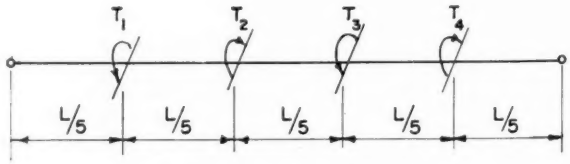

FIGURE 3. Bar in torsion.

If the loading is symmetrical, $T_1 = T_4$ and $T_2 = T_3$ then $T_1 = -T_2$ and $T_3 = -T_4$. The geometrical behaviour is expressed by

$$\omega_1 = \omega_{12} + \omega_2$$

where ω_1 is the angle of rotation at section one, ω_2 is the angle of rotation at section two and ω_{12} is the angle of twist, the rotation at section one relative to that at section two.

For a bar with a circular cross section the relative angle of twist may be expressed in terms of the torsional moment by means of the resistive factor G, a constant, and J, the polar moment of inertia of the cross section.

$$\omega_{12} = \omega_1 - \omega_2 = \frac{T_1 L}{5GJ} \quad . \quad . \quad (6)$$

METHOD

3. Identification

A sufficiently close identification of the geometrical and resistive behaviours at the joints of a structure is obtained by equating displacements and rotations only, at the intersections of the axes of the single members. A complete identification would require identification at all the points which make up an actual joint. Like the concentrated load, the joint becomes a concentrated or single-point joint. Identification establishes continuity of structure at the joints by equating displacements and rotations (both as vector quantities) of the ends of the members which meet at a joint.

Geometrical Identification—For any one set of single members to behave geometrically like the given structure, it is necessary and sufficient to equate the geometrical behaviour of set I (bars, bands or strings) to the geometrical behaviour of an intersecting set II. Geometrical behaviour is defined by the displacements $\boldsymbol{\rho}$ of a point of intersection and the derivatives of $\boldsymbol{\rho}$, thus

$$\left.\begin{aligned} &\boldsymbol{\rho}^{\mathrm{I}} = \boldsymbol{\rho}^{\mathrm{II}} \\ &\frac{\partial \boldsymbol{\rho}^{\mathrm{I}}}{\partial s_i} = \frac{\partial \boldsymbol{\rho}^{\mathrm{II}}}{\partial s_i}, \quad \frac{\partial^2 \boldsymbol{\rho}^{\mathrm{I}}}{\partial s_i \partial s_j} = \frac{\partial^2 \boldsymbol{\rho}^{\mathrm{II}}}{\partial s_i \partial s_j}, \quad (i, j = 1, 2) \end{aligned}\right\} \quad (7)$$

and for higher order derivatives.

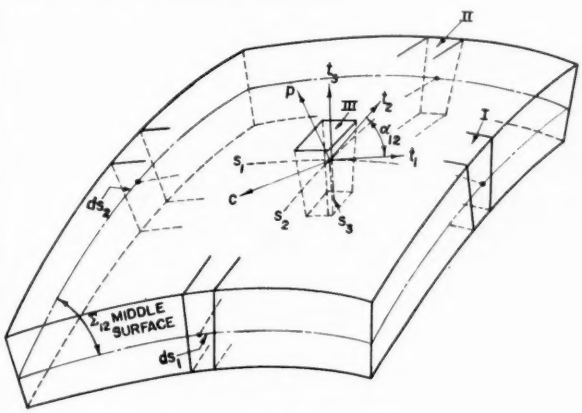

FIGURE 4. Shell-member.

These equations, along with the equations of integrability and compatibility where necessary, express the conditions of continuity of the structure from an undeformed state to a deformed state.

Resistive Identification—In the applications given the same material is assumed through the structure, hence it follows that the resistive behaviour I, is identical to the resistive behaviour II.

For the surface, or thin shell, -member, the identification is made for points in the reference surface only, as a sufficiently close approximation to a complete identification.

4. Conjugate Load Equations—Line-Member Structure

The analysis is completed by the statement that a load applied at a point in the structure is distributed to each member meeting at the point in such proportions that the algebraic sum of the distributed parts in any direction is equal to the component of the applied load in that direction. Since the part of an applied load which is carried by a member is related to the applied load, it is convenient to designate such a part as the conjugate load (on the member). For a load applied between joints the actual load is itself equal to the conjugate load on the member. At such a point all of the applied load is "distributed" to the member on which it acts.

Shell-member—Here the actual loads $\mathbf{p}$, force per unit area, and $\mathbf{c}$, couple per unit area, on the shell are distributed to the bands, I and II (Fig. 4) so that the sum of the conjugate loads equals the actual load.

$$\mathbf{p}^{\mathrm{I}} + \mathbf{p}^{\mathrm{II}} = \mathbf{p}, \qquad \mathbf{c}^{\mathrm{I}} + \mathbf{c}^{\mathrm{II}} = \mathbf{c}. \quad . \quad (8)$$

APPLICATIONS

5. Structures Composed of Line Members

In the examples given the joints in the structure are assumed to be either of the frictionless pinned type or the completely rigid type. In all cases, the resistive behaviour is assumed to be the same throughout the structure.

CONTINUOUS BEAM

A horizontal straight bar is supported over any number of supports and loaded by vertical forces (Fig. 5). The structure is considered as a combination of single line-members each of which extends from one support to an adjacent one and each member rigidly joined to its neighbour at the support.

The conjugate-load equations for the joint (n) are

Force, $R_{n,n} + R_{n+1,n} = R_n$ (first subscript denotes member, second denotes joint)

Couple, $M_{n,n} - M_{n+1,n} = C_n = 0$

Identification equations are:

Rotation, $\theta_{n,n} = \theta_{n+1,n}$

Displacement, $v_{n,n} = v_{n+1,n} = 0$ (by assumption).

The rotations are obtained from equations (3) and substituted into the identification equation for rotations. With the use of the conjugate-load equation for the bending couples the well known three-moment equation is obtained.

$$\frac{L_n}{I_n} M_{n,n-1} + 2\left[\frac{L_n}{I_n} + \frac{L_{n+1}}{I_{n+1}}\right] M_{n,n} + \frac{L_{n+1}}{I_{n+1}} M_{n+1,n+1} = -\frac{6}{L_n I_n} Q_{n,n-1} - \frac{6}{L_{n+1}, I_{n+1}} Q_{n+1,n+1}.$$

FRAME IN A VERTICAL PLANE

The frame shown in Fig. 6 with a load of 1,000 tons in the plane of the structure is an example given by R. V. Southwell[2] to illustrate his relaxation method. Member 3 is pinned at joints [1] and [3], otherwise the joints are rigid.

Area of Cross Section — *Moment of Inertia and Modulus of Elasticity*

$a_1 = a_2 = a_4 = 1$ in.^{2} $\quad I_1 = I_2 = I_3 = 1{\cdot}25$ in.^{4}

$a_3 = 1{\cdot}5$ in.^{2} $\quad E = 13{,}000$ tons/in.^{2}

Conjugate-Load Equations

Joint	*Force*	*Couple*
[2]	$X_{12} + X_{22} = 0$ $Y_{12} + Y_{22} = 0$	$M_{12} - M_{22} = 0$
[3]	$X_{23} + X_{33} + X_{43} = 0$ $Y_{23} + Y_{33} + Y_{43} = 0$	$M_{23} + M_{13} = 0$

Identification

Displacement	*Rotation*
$u_{12} = u_{22}$	$\theta_{12} = \theta_{22}$
$v_{12} = v_{22}$	
$u_{23} = u_{33} = u_{43}$	$\theta_{23} = \theta_{43}$
$v_{23} = v_{33} = v_{43}$	

Equilibrium Equations

Member

1 $\quad M_{11} - M_{12} + 50Y_{12} + 2{,}000 = 0$
2 $\quad M_{22} - M_{23} + 50X_{22} = 0$
4 $\quad M_{43} - M_{44} + 50Y_{43} = 0$

1	$X_{11} + X_{12} = 0$	$Y_{11} + Y_{12} - 1{,}000 = 0$
2	$X_{22} + X_{23} = 0$	$Y_{22} + Y_{23} = 0$
3	$X_{31} + X_{33} = 0$	$Y_{31} + Y_{33} = 0$
4	$X_{44} + X_{43} = 0$	$Y_{41} + Y_{43} = 0$

The following load-deformation relations for the members 1, 2, 3 and 4 are found from the equations (1), (2) and (3)

Member 1

$$X_{12} = 260u_{12}$$
$$Y_{12} = 352 - 39\theta_{12} + 1{\cdot}56v_{12}$$
$$M_{11} = -7{,}200 - 650\theta_{12} + 39v_{12}$$
$$M_{12} = -4{,}800 + 1{,}300\theta_{12} - 39v_{12}$$

Member 2

$$X_{22} = 39\theta_{22} + 39\theta_{23} + 1{\cdot}56u_{22} - 1{\cdot}56u_{23}$$
$$Y_{22} = -260v_{23} + 260v_{22}$$
$$M_{22} = -1{,}300\theta_{22} - 650\theta_{23} - 39u_{22} + 39u_{23}$$
$$M_{23} = 650\theta_{22} + 1{,}300\theta_{23} + 39u_{22} - 39u_{23}$$

Member 3

$$X_{33} = 138{\cdot}2u_{33} - 138{\cdot}2v_{33}$$
$$Y_{33} = -138{\cdot}2u_{33} + 138{\cdot}2v_{33}$$

Member 4

$$X_{43} = 260u_{23}$$
$$Y_{43} = -39\theta_{43} + 1{\cdot}56v_{43}$$
$$M_{43} = 1{,}300\theta_{43} - 39v_{43}$$
$$M_{44} = -650\theta_{43} + 39v_{43}$$

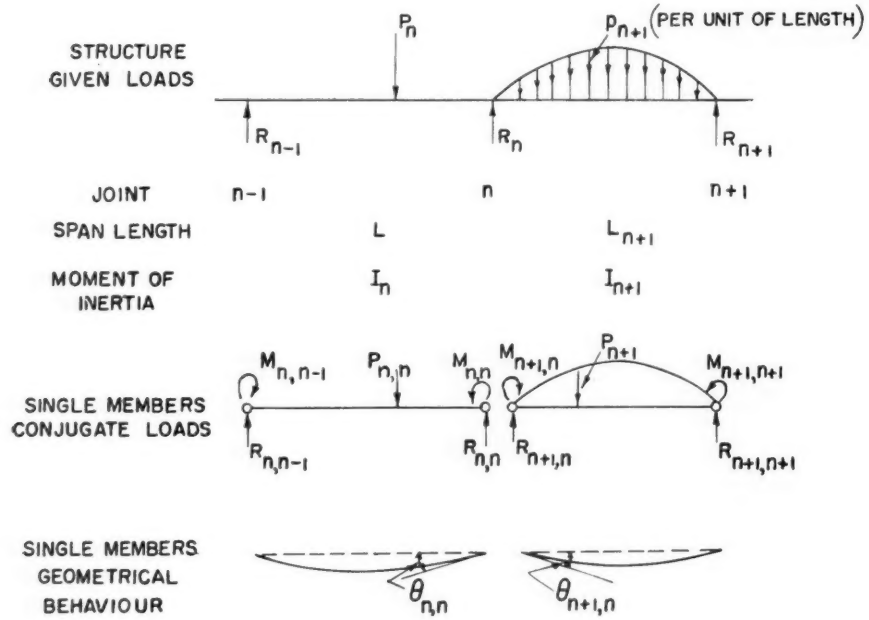

FIGURE 5. Continuous beam.

From the conjugate-load, identification and equilibrium equations the following equations are obtained:

$$261{\cdot}56u_{12}+39\theta_{12}-1{\cdot}56u_{23}+39\theta_{23}=0$$
$$261{\cdot}56v_{12}-39\theta_{12}-260v_{23}=-352$$
$$39u_{12}-39v_{12}+2{,}600\theta_{12}-39u_{23}+650\theta_{23}=4{,}800$$
$$-1{\cdot}56u_{12}+399{\cdot}76u_{23}-39\theta_{12}-138{\cdot}2v_{23}-39\theta_{23}=0$$
$$-260v_{12}-138{\cdot}2u_{23}+399{\cdot}76v_{23}-39\theta_{23}=0$$
$$39u_{12}+650\theta_{12}-39u_{23}-39v_{23}+2{,}600\theta_{23}=0.$$

The solutions are: $u_{12}=-0{\cdot}21$, $v_{12}=-3{\cdot}90$, $\theta_{12}=1{\cdot}91$, $u_{23}=-0{\cdot}87$, $v_{23}=-2{\cdot}92$ and $\theta_{23}=-0{\cdot}53$. The conjugate loads are then easily computed from the load-deformation equations.

PINNED GRID

The grid is a frame in a horizontal plane. Single members form two systems of bars extending from support to support in directions x and y (Fig. 7). The single members are assumed to be pinned at the joints, and at the supports. The structure is loaded by vertical loads P, at the joints, all of the same magnitude, thus making the structure one that is symmetrical in loading. Geometrical symmetry is postulated, with the areas of cross section and the moments of inertia as constants throughout the frame, as well as the modulus of elasticity of the material.

Conjugate-Load Equations		*Identification*
Joint	*Force*	*Displacements*
[11]	$X_{11}+Y_{11}=P$	$w_{11}{}^{x}=w_{11}{}^{y}$
[12]	$X_{12}+Y_{12}=P$	$w_{12}{}^{x}=w_{12}{}^{y}$
[21]	$X_{21}+Y_{21}=P$	$w_{21}{}^{x}=w_{21}{}^{y}$
[22]	$X_{22}+Y_{22}=P$	$w_{22}{}^{x}=w_{22}{}^{y}$

Load-displacement equations are found by means of the equations (5). They are

Members y

$$EI\ 750w_{11}{}^{x}=L^3(11X_{11}+17X_{12})$$
$$EI\ 750w_{12}{}^{x}=L^3(17X_{11}+28X_{12})$$
$$EI\ 750w_{21}{}^{x}=L^3(11X_{21}+17X_{22})$$
$$EI\ 750w_{22}{}^{x}=L^3(17X_{21}+28X_{22})$$

Members x

$$EI\ 750w_{11}{}^{y}=L^3(11Y_{11}+17Y_{21})$$
$$EI\ 750w_{12}{}^{y}=L^3(17Y_{11}+28Y_{21})$$
$$EI\ 750w_{21}{}^{y}=L^3(11Y_{12}+17Y_{22})$$
$$EI\ 750w_{22}{}^{y}=L^3(17Y_{12}+28Y_{22})$$

The combination of identification and conjugate-load equations leads to the following set

$$22X_{11}+17X_{12}+17X_{21}=28P$$
$$17X_{11}+39X_{12}+17X_{22}=28P$$
$$17X_{11}+39X_{21}+17X_{22}=45P$$
$$17X_{12}+17X_{21}+56X_{22}=45P$$

Solutions are $X_{11}=X_{22}=P/2$, $X_{12}=11P/39$, $X_{21}=28P/39$.

RIGID GRID

For this example the grid shown in Fig. 7 is assumed to have rigid interior joints and pins at the supports. The cross sections of the members are assumed circular and constant in area throughout. Single members are shown in Fig. 8.

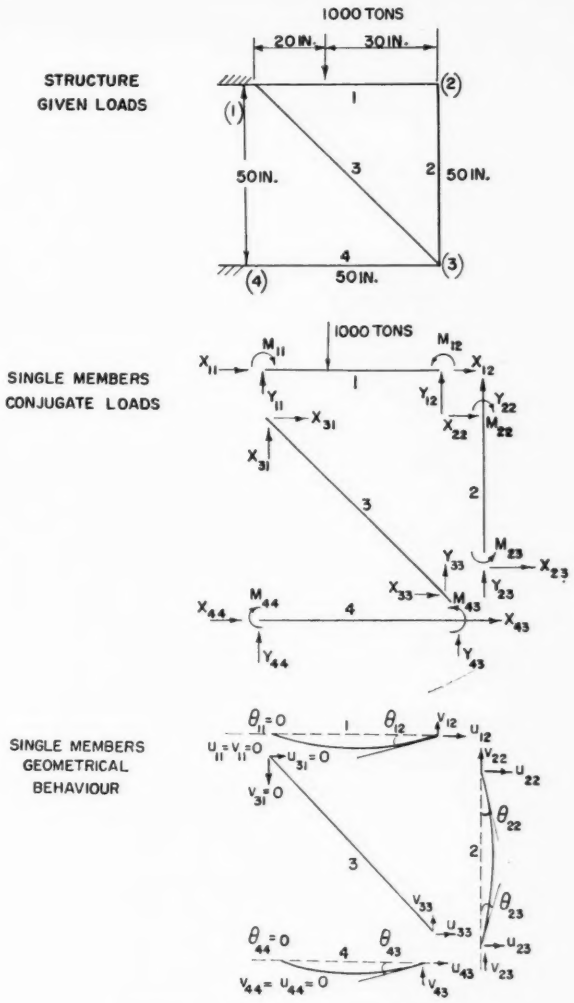

FIGURE 6. Frame in a vertical plane.

Conjugate-Load Equations			*Identification*	
Joint	*Force*	*Couple*	*Deflection*	*Rotation*
(ki)	$X_{ki}+Y_{ki}=P$,	$M_{ki}{}^{x}+T_{ki}{}^{y}=0$,	$w_{ki}{}^{x}=w_{ki}{}^{y}$	$\theta_{ki}{}^{x}=\omega_{ki}{}^{y}$
$(k,i)=(1,2,3,4)$		$M_{ki}{}^{y}+T_{ki}{}^{x}=0$		$\theta_{ki}{}^{y}+\omega_{ki}{}^{x}$

From the single member equations (4) and (5), the following equations are obtained:

Deflections

$$750EIw_{11}{}^{x}=(11X_{11}+17X_{12})L^3+(5M_{11}{}^{x}+6M_{12}{}^{x})5L^2$$
$$750EIw_{12}{}^{x}=(17X_{11}+28X_{12})L^3+(6M_{11}{}^{x}+11M_{12}{}^{x})5L^2$$
$$750EIw_{21}{}^{x}=(11X_{21}+17X_{22})L^3+(5M_{21}{}^{x}+6M_{22}{}^{x})5L^2$$
$$750EIw_{22}{}^{x}=(17X_{21}+28X_{22})L^3+(6M_{21}{}^{x}+11M_{22}{}^{x})5L^2$$
$$750EIw_{11}{}^{y}=(11Y_{11}+17Y_{21})L^3+(5M_{11}{}^{y}+6M_{21}{}^{y})5L^2$$
$$750EIw_{12}{}^{y}=(11Y_{12}+17Y_{22})L^3+(5M_{12}{}^{y}+6M_{22}{}^{y})5L^2$$
$$750EIw_{21}{}^{y}=(17Y_{11}+28Y_{21})L^3+(6M_{11}{}^{y}+11M_{21}{}^{y})5L^2$$
$$750EIw_{22}{}^{y}=(17Y_{12}+28Y_{22})L^3+(6M_{12}{}^{y}+11M_{22}{}^{y})5L^2$$

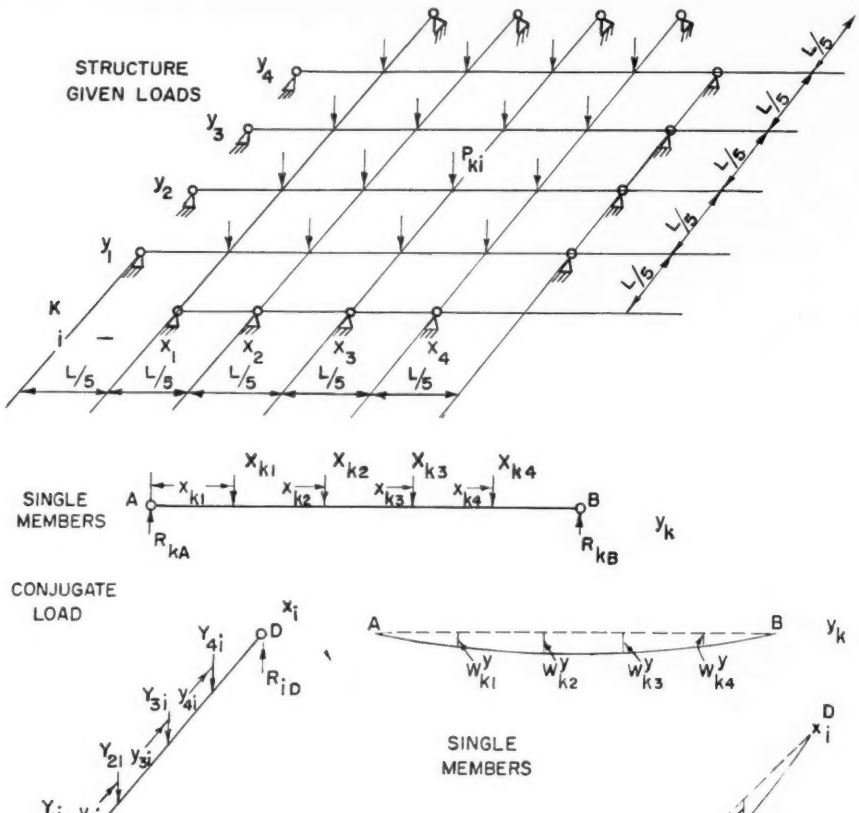

FIGURE 7. Pinned grid.

Rotations

$$-50EI\omega_{11}{}^{y}=3LX_{11}+5LX_{12}+5LM_{11}{}^{x}+10LM_{12}{}^{x}$$
$$-50EI\omega_{12}{}^{y}=LX_{11}+2LX_{12}+5LM_{12}{}^{x}$$
$$-50EI\omega_{21}{}^{y}=3LX_{21}+5LX_{22}+5LM_{21}{}^{x}+10LM_{22}{}^{x}$$
$$-50EI\omega_{22}{}^{y}=LX_{21}+2LX_{22}+5LM_{22}{}^{x}$$
$$50EI\omega_{11}{}^{x}=3LY_{11}+5LY_{21}-5LM_{11}{}^{y}-10LM_{21}{}^{y}$$
$$50EI\omega_{21}{}^{x}=LY_{11}+2LY_{21}-5LM_{21}{}^{y}$$
$$50EI\omega_{12}{}^{x}=3LY_{12}+5LY_{22}-5LM_{12}{}^{y}-10LM_{22}{}^{y}$$
$$50EI\omega_{22}{}^{x}=LY_{12}+2LY_{22}-5LM_{22}{}^{y}$$

From the conjugate-load equations for couples and symmetry of the torsional loading, the following equalities in bending moments are found

$$M_{11}{}^{x}=-M_{21}{}^{x},\ M_{22}{}^{x}=-M_{12}{}^{x},\ M_{11}{}^{y}=-M_{12}{}^{y}\ \text{and}\ M_{22}{}^{y}=-M_{21}{}^{y}$$

Equations (6) give four expressions for angles of twist:

$$5GJ(\omega_{11}{}^{x}-\omega_{12}{}^{x})=T_{11}{}^{x}L \qquad 5GJ(\omega_{11}{}^{y}-\omega_{21}{}^{y})=T_{11}{}^{y}L$$
$$5GJ(\omega_{21}{}^{x}-\omega_{22}{}^{x})=T_{21}{}^{x}L \qquad 5GJ(\omega_{12}{}^{y}-\omega_{22}{}^{y})=T_{12}{}^{y}L$$

The combination of the identification for deflections and the conjugate-load equations now leads to the following equations:

$$L(22X_{11}+17X_{12}+17X_{21})+25M_{11}{}^{x}+30M_{12}{}^{x}-25M_{11}{}^{y}-30M_{21}{}^{y}=28PL$$
$$L(17X_{11}+39X_{12}+17X_{22})+30M_{11}{}^{x}+55M_{12}{}^{x}-25M_{12}{}^{y}-30M_{22}{}^{y}=28PL$$
$$L(39X_{21}+17X_{22}+17X_{11})+25M_{21}{}^{x}+30M_{22}{}^{x}-30M_{11}{}^{y}-55M_{21}{}^{y}=45PL$$
$$L(17X_{21}+56X_{22}+17X_{12})+30M_{21}{}^{x}+55M_{22}{}^{x}-30M_{12}{}^{y}-55M_{22}{}^{y}=45PL$$

The combination of the identification equations for the rotations and the conjugate-load equations results in the following four equations:

$$L(3X_{11}-3X_{21}+5X_{12}-5X_{22})-2{\cdot}5M_{11}{}^{x}-20M_{22}{}^{x}=0$$
$$L(X_{11}+2X_{12}-X_{21}-2X_{22})+2{\cdot}5M_{22}{}^{x}=0$$
$$L(3X_{11}-3X_{12}+5X_{21}-5X_{22})+2{\cdot}5M_{11}{}^{y}+20M_{22}{}^{y}=0$$
$$L(X_{11}+2X_{21}-X_{12}-2X_{22})-2{\cdot}5M_{22}{}^{y}=0$$

Solutions of the eight preceding equations give the following results: By symmetry $X_{11}=X_{12}=P/2$. Then $X_{12}=414P/845$ and $X_{21}=431P/845$. Also

$$M_{22}{}^{x}=-M_{12}{}^{x}=M_{22}{}^{y}=-M_{21}{}^{y}=51PL/4{,}225$$

and

$$M_{11}{}^{x}=M_{11}{}^{y}=-M_{21}{}^{x}=-M_{12}{}^{y}=-544PL/4{,}225.$$

Deflections are:

$$750EI\,w_{11}=3{,}463PL^{3}/338$$
$$750EI\,w_{12}=29{,}899PL^{3}/1{,}690,\quad w_{12}=w_{21}$$
$$750EI\,w_{22}=22{,}982PL^{3}/845.$$

SHELL-MEMBER

The usual procedure in the analysis of the behaviour of a deformable body is to select an infinitesimal element of the deformed body, assume Hooke's Law for the material and express the equations which relate the loads to the displacements and their derivatives. These differential equations "of elasticity" are then subject to solution for whatever boundary conditions may be imposed on the body. In the case of a shell-structure an integration of stress over the thickness results in an element of the shell with one dimension finite. In the analysis which follows the element selected is a band with only one dimension infinitesimal. The geometry of the shell may then be defined in its deformed or undeformed state by either one of the two intersecting systems of bands, I and II. The behaviour of the shell is then based on the behaviour of a single band in either set.

Shell-behaviour now becomes the determination of the loads[3] (conjugate), that must be applied to a single band so that its behaviour as an isolated member is the same as its behaviour as an integral part of the shell. Since each band of system I intersects with a band of the system II in a region (or joint) III (Fig. 4), the identification of the geometrical and resistive behaviours for a member I, with the same behaviours for a member II, at every point of the region III, leads to the differential equations for the conjugate loads.

THE THIN-SHELL

In the application of the method to the thin shell, the following assumptions are made in order to simplify the partial behaviours:

(*a*) *Geometrical behaviour*—
1. Orthogonal curves are selected on the undeformed middle surface of the shell.
2. $\tan^{2}(\theta/2)$ is neglected compared to unity. θ is the rotation of the tangent $\mathbf{t}$ to an undeformed curve, to its deformed position $\mathbf{t}'$.
3. Small strains and small angle-changes.

(*b*) *Equilibrium behaviour*—
Small geodetic curvatures and small changes in the geodetic curvatures; hence equilibrium equations may be written for undeformed elements.

(*c*) *Resistive behaviour*—
1. Homogeneous, isotropic material; Hooke's law then expresses the relation between the stress-tensor and the strain-tensor.

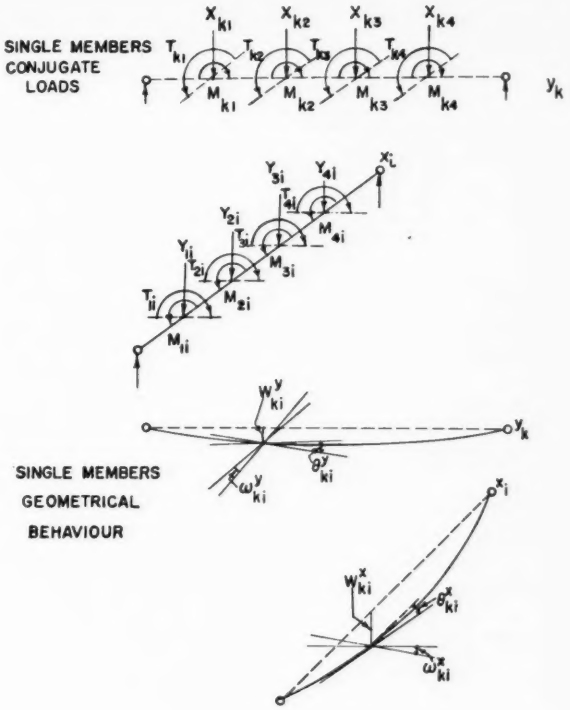

FIGURE 8. Rigid grid.

2. Stress σ_{12} is negligible compared to stresses σ_{23} and σ_{13}.

EQUATIONS OF EQUILIBRIUM

The equations of equilibrium, expressed in vector form, are (Fig. 9):

Band I

$$\left.\begin{aligned}&\mathbf{F}_{1}G_{21}+\frac{\partial\mathbf{F}_{1}}{\partial S_{1}}+\mathbf{p}^{\mathrm{I}}=0,\\&\mathbf{M}_{1}G_{21}+\frac{\partial\mathbf{M}_{1}}{\partial S_{1}}+\mathbf{F}_{1\wedge}\mathbf{t}_{1}+\mathbf{c}^{\mathrm{I}}=0\end{aligned}\right\}. \tag{9}$$

Band II

$$\left.\begin{aligned}&-\mathbf{F}_{2}G_{12}+\frac{\partial\mathbf{F}_{2}}{\partial S_{2}}+\mathbf{p}^{\mathrm{II}}=0,\\&-\mathbf{M}_{2}G_{12}+\frac{\partial\mathbf{M}_{1}}{\partial S_{2}}+\mathbf{F}_{2\wedge}\mathbf{t}_{2}+\mathbf{c}^{\mathrm{II}}=0\end{aligned}\right\}. \tag{10}$$

where,

$$\left.\begin{aligned}&\mathbf{F}_{1}\,dS_{2}=\int_{s_{3}}\boldsymbol{\sigma}_{23}\,da_{23},\quad \mathbf{M}_{1}\,dS_{2}=\int_{s_{3}}\mathbf{r}_{\wedge}\boldsymbol{\sigma}_{23}\,da_{23}\\&\mathbf{F}_{2}\,dS_{1}=\int_{s_{3}}\boldsymbol{\sigma}_{13}\,da_{13},\quad \mathbf{M}_{2}\,dS_{1}=\int_{s_{3}}\mathbf{r}_{\wedge}\boldsymbol{\sigma}_{13}\,da_{13}\end{aligned}\right\} \tag{10a}$$

THE RESISTIVE BEHAVIOUR

The resistive behaviour for the material is expressed by a linear relation between the stress-tensor $\bar{\sigma}$, and the strain-tensor, $\bar{e}$.

$$\bar{\sigma}dS = 2G\bar{e} + \lambda e_1 \bar{I} \quad . \quad . \quad (11)$$

where $G = \dfrac{E}{2(1+\mu)}$, $\lambda = \dfrac{\mu E}{(1+\mu)(1-2\mu)}$.

THE LATERALLY LOADED PLATE

A horizontal, rectangular plate is loaded with a distributed load p, normal to the middle surface, and distributed couples c_1 and c_2. Straight lines S_1 and S_2 are selected in the undeformed middle surface Σ_{12}, parallel to the edges of the plate. The curvatures (geodetic, torsional and normal) for S_1 and S_2 are, respectively, $G_{12}=G_{21}=0$, $T_{12}=T_{21}=0$, and $N_{12}=N_{21}=0$. Edge conditions are assumed such that the strain in the middle surface is zero.

To obtain the equilibrium equations in scalar form the following expressions are used:

$$\left.\begin{aligned}
\mathbf{F}_1 &= F_1\mathbf{t}_1 + F_{12}\mathbf{t}_2 + F_{13}\mathbf{t}_3 \\
\mathbf{F}_2 &= F_2\mathbf{t}_2 + F_{21}\mathbf{t}_1 + F_{23}\mathbf{t}_3 \\
\mathbf{M}_1 &= M_1\mathbf{t}_1 + M_{12}\mathbf{t}_2 + M_{13}\mathbf{t}_3 \\
\mathbf{M}_2 &= M_2\mathbf{t}_2 + M_{21}\mathbf{t}_1 + M_{23}\mathbf{t}_3 \\
\mathbf{p}^{\mathrm{I}} &= p_1{}^{\mathrm{I}}\mathbf{t}_1 + p_2{}^{\mathrm{I}}\mathbf{t}_2 + p_3{}^{\mathrm{I}}\mathbf{t}_3 \\
\mathbf{p}^{\mathrm{II}} &= p_1{}^{\mathrm{II}}\mathbf{t}_1 + p_2{}^{\mathrm{II}}\mathbf{t}_2 + p_3{}^{\mathrm{II}}\mathbf{t}_3 \\
\mathbf{c}^{\mathrm{I}} &= c_1{}^{\mathrm{I}}\mathbf{t}_1 + c_2{}^{\mathrm{I}}\mathbf{t}_2 + c_3{}^{\mathrm{I}}\mathbf{t}_3 \\
\mathbf{c}^{\mathrm{II}} &= c_1{}^{\mathrm{II}}\mathbf{t}_1 + c_2{}^{\mathrm{II}}\mathbf{t}_2 + c_3{}^{\mathrm{II}}\mathbf{t}_3
\end{aligned}\right\} \quad . \quad (12)$$

and $$\mathbf{r} = r_1\mathbf{t}_1 + r_2\mathbf{t}_2 + r_3\mathbf{t}_3.$$

With moments M_{13} and M_{23} assumed zero, the scalar equations of equilibrium are

Band I—axis S_1

$$\left.\begin{aligned}
\frac{\partial F_1}{\partial S_1} &= -p_1{}^{\mathrm{I}} & \frac{\partial M_1}{\partial S_1} &= -c_1{}^{\mathrm{I}} \\
\frac{\partial F_{12}}{\partial S_1} &= -p_2{}^{\mathrm{I}} & \frac{\partial M_{12}}{\partial S_1} + F_{12} &= -c_2{}^{\mathrm{I}} \\
\frac{\partial F_{13}}{\partial S_1} &= -p_3{}^{\mathrm{I}} & F_{12} &= -c_3{}^{\mathrm{I}} = 0
\end{aligned}\right\} \quad . \quad (13)$$

Band II—axis S_2

$$\left.\begin{aligned}
\frac{\partial F_2}{\partial S_2} &= -p_2{}^{\mathrm{II}} & \frac{\partial M_2}{\partial S_2} &= -c_2{}^{\mathrm{II}} \\
\frac{\partial F_{21}}{\partial S_2} &= -p_1{}^{\mathrm{II}} & \frac{\partial M_{21}}{\partial S_2} - F_{23} &= -c_1{}^{\mathrm{II}} \\
\frac{\partial F_{23}}{\partial S_2} &= -p_3{}^{\mathrm{II}} & F_{21} &= -c_3{}^{\mathrm{II}} = 0
\end{aligned}\right\} \quad . \quad (14)$$

Force and moment expressions in terms of geometrical behaviour are obtained from expressions (10a) by means of the stress-strain relation (11). The stress-vector is found from the scalar product of the unit-vector $\mathbf{t}$, and the stress-tensor $\bar{\sigma}$. For all directions $\mathbf{t}$

$$\sigma da = t \,.\, \bar{\sigma} dS$$

where . indicates a scalar product.

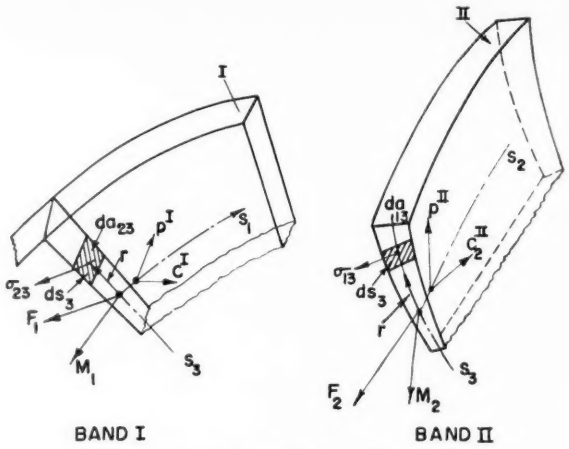

FIGURE 9. Elements of a shell-member.

The scalar components of force $\mathbf{F}$ are

$$\left.\begin{aligned}
F_1 &= \varepsilon_1 + \mu\varepsilon_2 = 0, \\
F_2 &= \varepsilon_2 + \mu\varepsilon_1 = 0 \\
F_{12} &= -\frac{1-\mu}{2} D\Delta\alpha_{12} = 0, \quad F_{21} = -\frac{1-\mu}{2} D\Delta\alpha_{12} = 0 \\
F_{13} &= -\frac{1-\mu}{2} D\Delta\alpha_{13} + \frac{K\mu}{4}\frac{\partial \overline{A}}{\partial S_1}, \\
F_{23} &= -\frac{1-\mu}{2} D\Delta\alpha_{23} + \frac{K\mu}{4}\frac{\partial \overline{A}}{\partial S_2}.
\end{aligned}\right\} \quad (15)$$

The scalar components of the moment $\mathbf{M}$ are

$$\left.\begin{aligned}
M_{12} &= K(\Delta N_{12} + \mu\Delta N_{21}), \\
M_{21} &= -K(\Delta N_{21} + \mu\Delta N_{12}) \\
M_1 &= \frac{K}{2}(1-\mu)(\Delta T_{12} - \Delta T_{21}), \\
M_2 &= -\frac{K}{2}(1-\mu)(\Delta T_{12} - \Delta T_{21})
\end{aligned}\right\} \quad (16)$$

where

$$D = \frac{Eh}{1-\mu^2}, \qquad K = \frac{Eh^3}{12(1-\mu^2)}, \qquad \overline{A} = -(\Delta N_{12} + \Delta N_{21}).$$

With $p_1 = p_2 = 0$, $p_1{}^{\mathrm{II}} = p_2{}^{\mathrm{I}} = 0$, and $p_1{}^{\mathrm{I}} = p_2{}^{\mathrm{II}} = 0$, the equilibrium equations combined with the force and moment expressions, result in

$$p_3{}^{\mathrm{I}} - \frac{\partial c_2}{\partial S_1} = K\frac{\partial \Delta N_{12}}{\partial S_1{}^2} + \mu K\frac{\partial^2 \Delta N_{21}}{\partial S_1{}^2} \quad . \quad (17)$$

$$-p_3{}^{\mathrm{II}} - \frac{\partial c_1{}^{\mathrm{II}}}{\partial S_2} = -K\frac{\partial^2 \Delta N_{21}}{\partial S_2{}^2} - \mu K\frac{\partial^2 \Delta N_{21}}{\partial S_2{}^2} \quad (18)$$

$$\frac{\partial c_2{}^{\mathrm{II}}}{\partial S_1} = -\frac{\partial c_1{}^{\mathrm{I}}}{\partial S_2} = -K(1-\mu)\frac{\partial^2 \Delta N_{12}}{\partial S_2{}^2} = -K(1-\mu)\frac{\partial^2 \Delta N_{21}}{\partial S_1{}^2} \quad . \quad . \quad (19)$$

Differentiation and combination of equations (17) and (18), along with the integrability condition, leads to

$$\left.\begin{aligned} K(1-\mu^2)\frac{\partial^4 \Delta N_{12}}{\partial S_1^{\,2}\partial S_2^{\,2}} &= \frac{\partial^2 p_3^{\,\mathrm{I}}}{\partial S_2^{\,2}} - \mu\frac{\partial^2 p_3^{\,\mathrm{II}}}{\partial S_1^{\,2}} - \\ &\quad - \frac{\partial^3 c_1^{\,\mathrm{I}}}{\partial S_2^{\,2}\partial S_1} - \mu\frac{\partial^3 c^{\mathrm{II}}{}_1}{\partial S_1^{\,2}\partial S_2} \\ K(1-\mu^2)\frac{\partial^4 \Delta N_{21}}{\partial S_2^{\,2}\partial S_1^{\,2}} &= \frac{\partial^2 p_3^{\,\mathrm{II}}}{\partial S_1^{\,2}} - \mu\frac{\partial^2 p_3^{\,\mathrm{I}}}{\partial S_2^{\,2}} + \\ &\quad + \frac{\partial^3 c_1^{\,\mathrm{II}}}{\partial S_1^{\,2}\partial S^2} + \mu\frac{\partial^3 c_2^{\,\mathrm{I}}}{\partial S_2^{\,2}\partial S_1} \end{aligned}\right\} . \quad (20)$$

From (19)

$$\left.\begin{aligned} K(1-\mu^2)\frac{\partial^4 \Delta N_{12}}{\partial S_1^{\,2}\partial S_2^{\,2}} &= (1+\mu)\frac{\partial^3 c_1^{\,\mathrm{I}}}{\partial S_1^{\,2}\partial S_2} \\ K(1-\mu^2)\frac{\partial^4 \Delta N_{21}}{\partial S_2^{\,2}\partial S_1^{\,2}} &= -(1+\mu)\frac{\partial^3 c_2^{\,\mathrm{II}}}{\partial S_2^{\,2}\partial S_1} \end{aligned}\right\} . \quad (21)$$

Since $M_1^{\,\mathrm{I}} = -M_2^{\,\mathrm{II}}$ and with substitution $c_1^{\,\mathrm{II}} = c_1 - c_1^{\,\mathrm{I}}$, $c_2^{\,\mathrm{I}} = c_2 - c_2^{\,\mathrm{II}}$, equations (20) and (21) become

$$\Delta\frac{\partial c_2^{\,\mathrm{II}}}{\partial S_1} = -\frac{\partial^2 p_3^{\,\mathrm{I}}}{\partial S_2^{\,2}} + \mu\frac{\partial^2 p_3^{\,\mathrm{II}}}{\partial S_1^{\,2}} + \frac{\partial^3 c_2}{\partial S_2^{\,2}\partial S_1} + \mu\frac{\partial^3 c_1}{\partial S_1^{\,2}\partial S_2} \quad (22)$$

$$\Delta\frac{\partial c_2^{\,\mathrm{II}}}{\partial S_1} = -\frac{\partial^2 p_3^{\,\mathrm{II}}}{\partial S_1^{\,2}} + \mu\frac{\partial^2 p_3^{\,\mathrm{I}}}{\partial S_2^{\,2}} - \frac{\partial^3 c_1}{\partial S_1^{\,2}\partial S_2} - \mu\frac{\partial^3 c_2}{\partial S_2^{\,2}\partial S_1} \quad (23)$$

where Δ is the operator,

$$\frac{\partial^2}{\partial S_1^{\,2}} + \frac{\partial^2}{\partial S_2^{\,2}} .$$

Subtraction of equations (22) and (23) with $p_3^{\,\mathrm{I}} = p_3 - p_3^{\,\mathrm{II}}$, results in

$$\Delta p_3^{\,\mathrm{II}} = \frac{\partial^2 p_3}{\partial S_2^{\,2}} - \frac{\partial^3 c_1}{\partial S_1^{\,2}\partial S_2} - \frac{\partial^3 c_2}{\partial S_2^{\,2}\partial S_1} . \quad (24)$$

From (22) and (24)

$$\frac{1}{1-\mu}\Delta\frac{\partial c_2^{\,\mathrm{II}}}{\partial S_1} = -\frac{\partial^2 p_3^{\,\mathrm{II}}}{\partial S_1^{\,2}} - \frac{\partial^3 c_1}{\partial S_1^{\,2}\partial S_2} . \quad (25)$$

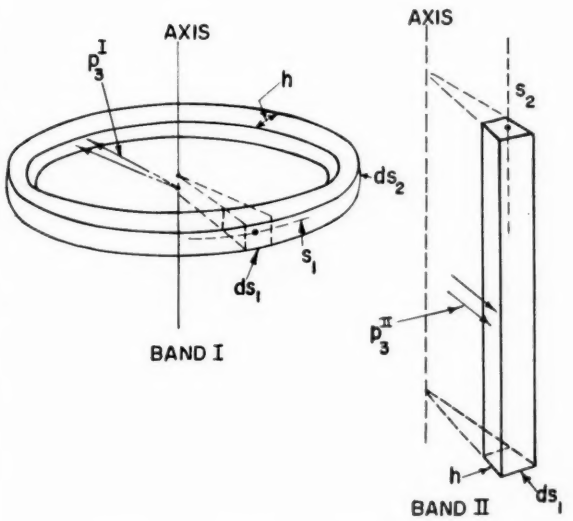

FIGURE 10. Elements of a cylindrical shell.

Equations (24) and (25) are the conjugate-load equations for the bands II. Equations for Bands I are obtained by the substitutions

$$p_3^{\,\mathrm{II}} = p_3 - p_3^{\,\mathrm{I}} \text{ and } c_2^{\,\mathrm{II}} = c_2 - c_2^{\,\mathrm{I}} . \quad (8)$$

The usual plate-equation[4] is found by subtracting equations (17) and (18). In terms of the lateral displacement w, the result is

$$K\left(\frac{\partial^4 w}{\partial S_1^{\,4}} + 2\frac{\partial^4 w}{\partial S_1^{\,2}\partial S_2^{\,2}} + \frac{\partial^4 w}{\partial S_2^{\,4}}\right) = -p_3 + \frac{\partial c_2}{\partial S_1} - \frac{\partial c_1}{\partial S_2} \quad (26)$$

where $K = \dfrac{Eh^3}{12(1-\mu^2)}$.

CYLINDRICAL TANK OF CONSTANT THICKNESS

A circular cylindrical tank is subjected to a distributed load p_3 normal to the middle surface and symmetrical with respect to the axis of the tank. Bands I (Fig. 10) are bounded by S_1-lines obtained by planes perpendicular to the axis of the undeformed tank. Bands II are bounded by S_2-lines in radial planes which contain the axis of the tank.

Because of symmetry of load and structure, all derivatives with respect to S_1 are zero. Curvatures $N_{21} = T_{12} = T_{21} = 0$. It is assumed that the weight of the tank is negligible, hence forces F_2 are zero. Changes in torsional curvature ΔT_{12}, are also zero under the conditions assumed. It follows that $M_1 = M_2 = 0$ and for $c_1 = c_2 = 0$, $c_1^{\,\mathrm{I}} = c_1^{\,\mathrm{II}} = c_2^{\,\mathrm{I}} = c_2^{\,\mathrm{II}} = 0$. For $p_1 = p_2 = 0$, $p_1^{\,\mathrm{I}} = p_1^{\,\mathrm{II}} = p_2^{\,\mathrm{I}} = p_2^{\,\mathrm{II}} = 0$.

The equations of equilibrium (9) and (10) reduce to

$$F_1 N_{12} = p_3^{\,\mathrm{I}}, \qquad -\frac{\partial F_{23}}{\partial S_2} = -p_3^{\,\mathrm{II}}, \qquad \frac{\partial M_{21}}{\partial S_2} = F_{23} . \quad (27)$$

The force and moment expressions are

$$\left.\begin{aligned} F_1 &= D(\varepsilon_1 + \mu\varepsilon_2) + KN_{12}\left(\frac{\overline{\mu}\,\overline{A}}{2} - \Delta N_{12}\right) \\ F_2 &= D(\varepsilon_2 + \mu\varepsilon_1) + KN_{12}\left(\frac{\overline{\mu}\,\overline{A}}{2} + \Delta N_{21}\right) \end{aligned}\right\} . \quad (28)$$

$$\left.\begin{aligned} M_{12} &= K(\Delta N_{12} + \mu\Delta N_{21} - \varepsilon_0 N_{12}) \\ M_{21} &= K[N_{12}(\varepsilon_2 + \mu\varepsilon_1) + \Delta N_{21} + \mu\Delta N_{12} - \mu\varepsilon_0 N_{12}] \end{aligned}\right\} \quad (29)$$

in which $\overline{A} = -\Delta N_{12} - \Delta N_{21}$

$$\varepsilon_0 = \overline{\mu}(\varepsilon_1 + \varepsilon_2), \ \overline{\mu} = \frac{\mu}{1-\mu} .$$

Changes in normal curvature in terms of deflections are:

$$\Delta N_{12} = -wN_{12}^{\,2}, \qquad \Delta N_{21} = -\frac{\partial^2 w}{\partial S_2^{\,2}} . \quad (30)$$

By differentiation it follows that

$$\frac{\partial^2 \Delta N_{12}}{\partial S_2^{\,2}} = -N_{12}^{\,2}\frac{\partial^2 w}{\partial S_2^{\,2}} = N_{12}^{\,2}\Delta N_{21} . \quad (31)$$

From the strain-displacement equations

$$\varepsilon_1 = wN_{12} = -\frac{\Delta N_{12}}{N_{12}} . \quad (32)$$

Elimination of ε_2 and the substitution of ε_1 from equation (32) into the equations (28) results in

$$F_1 N_{12} = -k_1 \Delta N_{12} - K N_{12}^2 \frac{\mu}{2}(3+\mu)\Delta N_{21} \qquad (33)$$

in which

$$k_1 = (1-\mu^2)D - K N_{12}^2 \left[1 + \frac{\bar{\mu}}{2}(1-\mu^2)\right]$$

The moment M_{21} becomes

$$M_{21} = K k_2 \Delta N_{21} + \mu K k_3 \Delta N_{12} \qquad (34)$$

where

$$k_2 = 1 - \frac{K}{D} N_{12}^2 (1-\mu\bar{\mu})\left(1 - \frac{\mu\bar{\mu}}{2}\right)$$

$$k_3 = 1 + \mu + \frac{K}{D} N_{12}^2 \frac{\mu\bar{\mu}}{2}\left(1 - \mu\bar{\mu}\right).$$

THE LOAD-EQUATIONS

Loads may now be expressed in terms of changes in curvature.

$$p_3^{\mathrm{I}} = N_{12} F_1 = -k_1 \Delta N_{12} - K N_{12}^2 \frac{\mu}{2}(3+\mu)\Delta N_{21} \qquad (35)$$

$$p_3^{\mathrm{II}} = -\frac{\partial^2 M_{21}}{\partial S_2^2} = -k_2 K \frac{\partial^2 \Delta N_{21}}{\partial S_2^2} - k_3 K \mu N_{12}^2 \Delta N_{21} \qquad (36)$$

$$\frac{\partial^2 p_3^{\mathrm{I}}}{\partial S_2^2} = -k_1 N_{12}^2 \Delta N_{21} - K N_{12}^2 \frac{\mu}{2}(3+\mu)\frac{\partial^2 \Delta N_{21}}{\partial S_2^2} \qquad (37)$$

$$\frac{\partial^2 p_3^{\mathrm{II}}}{\partial S_2^2} = -k_2 K \frac{\partial^4 \Delta N_{21}}{\partial S_2^4} - \mu k_3 K N_{12}^2 \frac{\partial^2 \Delta N_{21}}{\partial S_2^2} \qquad (38)$$

$$\frac{\partial^4 p_3^{\mathrm{I}}}{\partial S_2^4} = -k_1 N_{12}^2 \frac{\partial^2 \Delta N_{21}}{\partial S_2^2} - K N_{12}^2 \frac{\mu}{2}(3+\mu)\frac{\partial^4 \Delta N_{21}}{\partial S_2^4} \qquad (39)$$

If ΔN_{21} and its derivatives are eliminated from equations (36), (37), (38) and (39) the following differential equation for conjugate loads is obtained:

$$k_2 K \frac{\partial^4 p_3^{\mathrm{I}}}{\partial S_2^4} - K N_{12}^2 \frac{\mu}{2}(3+\mu)\frac{\partial^2 p_3^{\mathrm{II}}}{\partial S_2^2} + k_3 K \mu N_{12}^2 \frac{\partial^2 p_3^{\mathrm{I}}}{\partial S_2^2} - k_1 N_{12}^2 p_3^{\mathrm{II}} = 0. \qquad (40)$$

With the substitution

$$p_3^{\mathrm{I}} = p_3 - p_3^{\mathrm{II}}$$

$$k_2 K \frac{\partial^4 p_3^{\mathrm{II}}}{\partial S_2^4} + K N_{12}^2 \mu [k_3 + (3+\mu)/2]\frac{\partial^2 p_3^{\mathrm{II}}}{\partial S_2^2} + k_1 N_{12}^2 p_3^{\mathrm{II}} = k_3 K N_{12}^2 \mu \frac{\partial^2 p_3}{\partial S_2^2} + k_2 K \frac{\partial^4 p_3}{\partial S_2^4} \qquad (41)$$

If the k-values are approximated by $k_1 = D(1-\mu^2)$, $k_2 = 1$, and $k_3 = 1+\mu$, equation (41) simplifies into

$$K \frac{\partial^4 p_3^{\mathrm{II}}}{\partial S_2^4} + K N_{12}^2 \frac{\mu}{2}(5+3\mu)\frac{\partial^2 p_3^{\mathrm{II}}}{\partial S_2^2} + D(1-\mu^2) N_{12}^2 p_3^{\mathrm{II}} = \mu K N_{12}^2 (1+\mu)\frac{\partial^2 p_3}{\partial S_2^2} + K \frac{\partial^4 p_3}{\partial S_2^4}. \qquad (42)$$

EQUATION FOR DEFLECTIONS

Equations (35) and (36) may be added. Then

$$p_3^{\mathrm{I}} + p_3^{\mathrm{II}} = p_3 = -k_1 \Delta N_{12} - K N_{12}^2 \mu \left[k_3 + \frac{3+\mu}{2}\right]\Delta N_{21} - k_2 K \frac{\partial^2 \Delta N_{21}}{\partial S_2^2} \qquad (43)$$

With $\Delta N_{12} = -w N_{12}^2$

and $\Delta N_{21} = -\dfrac{\partial^2 w}{\partial S_2^2}$,

the equation for p_3 becomes

$$p_3 = k_1 N_{12}^2 w + K N_{12}^2 \mu \left(k_3 + \frac{3+\mu}{2}\right)\frac{\partial^2 w}{\partial S_2^2} + k_2 K \frac{\partial^4 w}{\partial S_2^4}. \qquad (44)$$

For approximate k-values, the equation for p_3 is

$$K \frac{\partial^4 w}{\partial S_2^4} + K N_{12}^2 \frac{\mu}{2}(5+3\mu)\frac{\partial^2 w}{\partial S_2^2} + D N_{12}^2 (1-\mu^2) w = p_3. \qquad (45)$$

If the middle term on the left hand side is small in comparison to the other terms and is neglected, then the usual tank equation[5] is obtained.

$$K \frac{\partial^4 w}{\partial S_2^4} + D N_{12}^2 (1-\mu^2) w = p_3. \qquad (46)$$

REFERENCES

1. WESTERGAARD, H. M. (1921). Deflection of Beams by the Conjugate Beam Method. *Journal Western Soc. Engineers*, Vol. 26, p. 368, 1921.
2. SOUTHWELL, R. V. (1940). *Relaxation Methods in Engineering Science*, p. 48, 1940.
3. Trial-load Method of Analyzing Arch-Dams, U.S. Dept. Interior Bureau of Reclamation (1938), Part V—Tech. Invest.—Bulletin 1, 1938.
   Note: "Trial-loads" are successive approximations to the conjugate-loads.
4. TIMOSHENKO, S. (1940). *Theory of Plates and Shells*, p. 88, 1940.
5. FLÜGGE, W. (1934). *Statik u. Dynamik der Schalen*, p. 131, 1934.

The Development of the Aircraft Industry in the Union of Soviet Socialist Republics

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Introduction

Progressively less information becomes available from the Soviet Union about the development of its aircraft industry. In examining the material that has been released during the past five decades, much confusion arises in the initial attempts to piece this material together chronologically and to show realistically the development of the aircraft industry in the Union of Soviet Socialist Republics. It seems that much time would be spent before a somewhat realistic picture might be obtained. Therefore, it appears appropriate that a definitive analysis be presented at this time.

The industrial development under the Communists in the Soviet Union is, and has been, a direct result of a bureaucratic state-planned economy. Voznesensky points out* that the balance of the national economy, which lies in the hands of the State, includes the production and allocation of the social product, the production and allocation of fixed capital, the balance and distribution of man-power, the production and allocation of national income, the balance of money incomes and outlays of the population, and the balance and allocation of material resources. There is no doubt that the Soviets are placing their future hopes of industrial development, as they have done in the past, on a series of planned economic programmes. Vast amounts of statistical data are continuously collected and examined; this data serves as a basis for the projected Five-Year Plans. Some amazing industrial developments have been achieved in the past, but these developments have usually occurred at the expense of increasing the standard of living as well as the "freedom of the individual."

The aviation industry was all but obliterated in Russia by the closing year of the revolution. Since most of the engineers and technicians had either been eradicated or exiled, it was not long before the Soviets were importing both machines and "brains" from Great Britain, France, Italy, and Germany. Soon after the First World War exceptionally cordial relations existed between Germany and Soviet Russia†; since the German industrial and aeronautical output was limited by the Allies after the First World War, many Germans went to the U.S.S.R. to make use of their aeronautical experience and to aid in the development of the Soviet aeronautical industry. The impact of these foreigners, primarily Germans, can be traced throughout the development of the industry. The general philosophy towards the aircraft industry and the role of the military air power, which had been inherent in Germany throughout the war years, was transplanted to the U.S.S.R. to form the base of the Soviet philosophy towards aeronautics. Thus, the Soviet military air power was developed as a tactical aid to the ground forces and actually remained subordinate to the ground army throughout the Second World War. Similarly, immediately after the First World War, both the British and the Germans felt that hydroplanes would play a leading role in the future of aviation. This influence was deeply felt in the Soviet Union during the early stages of its aeronautical development and much time and energy were spent on the development of hydroplanes and air sea routes.

It should be pointed out that immediately after the Second World War the Soviets did not disarm and reduce their defence expenditures as did the British and the Americans. This is shown by the graph in Fig. 1. Although American Lend-Lease aid to the U.S.S.R. ceased in August 1945, this was more than compensated for by the mass movement of German industry and "brains" to the Soviet Union. As soon as the war was over, the Soviets rebuilt installations such as the Peenemunde German Guided Missiles Centre, continuing the development and research work the Germans had initiated, not only with the same German installations but with the same German technicians and scientists as well. Since 1946, there has been a reorganisation of the Soviet Air Force, with due recognition being given to the construction of the strategic air arm. Although the greatest percentage of the aircraft output from 1946 to 1952 has apparently been of the defence and attack-fighter type of craft, there is no doubt that the planning and development of strategic aircraft, such as long-range inter-continental bombers and missiles, have progressed and units of this type will be in much greater production in the coming, and future, years.

Early Soviet Development

Although Czarist Russia had some brilliant pioneers in aerodynamics and aircraft design, the aircraft industry was only slightly developed and most of the flying machines were either imported or manufactured at home under foreign licence. From the best reports, about

*From *The Economy of the U.S.S.R. During World War II* by Nikolai A. Voznesensky, Deputy Premier of the U.S.S.R. and Chief of the State Planning Commission in 1947.

†On 16th April 1922 Soviet Russia signed her first treaty with a major power. This treaty is known as the "Treaty of Rapallo" and Germany and the Soviet Union not only established diplomatic relations between themselves but also renounced financial claims upon each other. Other powers, fearing a Soviet-German *entente*, were forced to a more conciliatory attitude towards both nations after this treaty.

Originally received 28th August 1953.

Revised 16th February 1954.

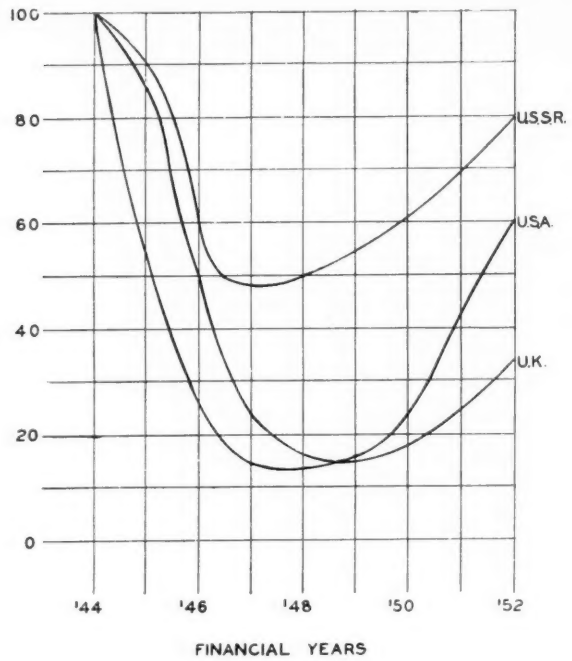

FIGURE 1. Defence expenditures. 1944 = 100.

The Soviet budget for 1952 allocated about 114 billion roubles or 23·9 per cent. of the total expenditures for "Defence." In the budget, the heading "Defence" covers only the direct expenditures of the service ministers on the maintenance of the armed forces and the purchase of their arms at cost prices. "Grants to the National Economy" include the construction of military installations, armament factories, aluminium factories, and so on. It is also quite clear, from Soviet statements, that the development of atomic energy is paid out of the "Social and Cultural" funds, and the cost of the large paramilitary forces (maintained by the Ministry of the Interior—M.V.D.) appears under "Administration and Justice." Thus, in reality, it is reasonable to assume that defence spending is at present exceeding one-third of the entire budget, the budget covering about two-thirds of the total Soviet national income.

1,770 aircraft and 660 aero-engines were produced in Czarist Russia during 1916. According to Baranov, output fell to 137 aircraft and 77 engines in 1919. By the end of the revolution, the aircraft industry in the newly formed Soviet Union was virtually non-existent and most of the aeronautical technicians and engineers had either been annihilated or had left the country.

Lenin, influenced by A. N. Tupolev and N. E. Zhukovski, was soon convinced of the importance of the aviation industry in the Soviet Union and, in 1918, Tupolev and Zhukovski founded the "ZAGI" (Central Aerodynamics and Hydrodynamics Research Institute) in Moscow. While Zhukovski spent most of his time with the problems of establishing a school and educating engineers, Tupolev was forced to divide his time between the educational arm of the "ZAGI," the "Zhukovski Air Academy," and the actual process of building aircraft. In 1922, Tupolev built the first all-Soviet flying machine, the ANT-1, a small single-engined monoplane constructed almost entirely of wood.

In order to build an aeronautical fleet, the early policy of the Soviet government was unavoidable. Foreign aircraft and foreign technicians, mechanics, and engineers had to be imported into the country. During the first six months of 1924, the Soviets purchased about 700 then-modern aircraft in Italy and Holland. The air programme for 1925 provided for the acquisition of 1,030 new aeroplanes, of which 500 were to be built in the Soviet Union, 200 were to be obtained from several Italian firms, and 330 were to be ordered from Holland.

Hugo Junkers, one of the foremost German designers and producers of aircraft, came to the Soviet Union in 1922 and built the first modern aircraft plant, the Fili plant near Moscow. Junkers, like the other German advisors, left a marked influence on the early Soviet aeronautical development. Tupolev, who worked with Junkers in these early years and later became the outstanding Soviet aircraft designer, incorporated many recognisably German characteristics in his bombers and other aeroplanes. Italian, British, and French influences were also apparent, although to a lesser degree than the German influences, during these early years. In both Leningrad and Moscow, Renault and Gnôme-Rhone aircraft engines were assembled between 1922 and 1925. Although the productivity of the Soviet workers was low, machines were scarce, the early plants and organisations were inefficient, and many technical personnel had to be imported from foreign countries, the Soviet aircraft industry had been initiated and aeronautical factories were producing flying machines by 1922.

By 1926, the entire aircraft industry was in the hands of the "State" and there were about twenty aircraft factories. There were also two factories producing the Soviet aluminium alloy "Koltjugalljuminii"—the Koltzchuquino Works at Vladimir and the Krassini Wyborschetz in Leningrad. By 1926, three all-Soviet aeroplanes had been designed and constructed—the ANT-1, a small single-engined monoplane fitted with a Blackburn "Tom-Tit" engine; the ANT-2, a three-seat commercial monoplane with a Bristol "Lucifer" engine; and the ANT-3*, a two-seat biplane with a Napier "Lion" engine.

The First Five-Year Plan, 1928-1932

The first Five-Year Plan, from 1928 to 1932, gave the Soviets the opportunity to expand their internal aircraft industry as well as their basic commodity output. Heavy industry and increased basic commodity production was emphasised during this five-year period. Although the first Five-Year Plan was less successful than the following Five-Year Plans and continual revisions were deemed necessary, great strides were made in the industrialisation of the comparatively agricultural Soviet Russia. In 1929, aluminium, which previously had to

*M. Gromoff and M. Radzevitch flew from Moscow to Konigsberg, Berlin, Paris, Rome, Vienna, Warsaw, and back to Moscow, a distance of 4,042 miles in thirty-four and a quarter hours flying time, averaging about 120 miles per hour, in an early ANT-3.

be imported from Germany, was first produced in the Soviet Union. By 1932, an annual production of approximately 1,500 aircraft was reached by the Soviet factories. Though such a figure may seem inconsequential in comparison to similar figures for the western countries, it represented a great advancement in comparison with earlier Soviet figures.

In 1932, the Soviet aircraft production was controlled by two distinct organisations. The production of military aircraft and equipment was controlled by the "War Aircraft Trust," which was directly under the "War Commissary," the latter corresponding to the German "Reichswehr Ministry." This "Trust" included airframe factories, aero-engine factories, aircraft accessory plants, experimental plants, repair shops, and a special section devoted to the centralisation of calculations, designs, construction, and assembly of aircraft. The "Civil Aircraft Trust," attached to the "Labour and Defence Council," was formed as a separate entity in October 1930, in order to lessen the burden on the military aircraft industry. The "Civil Aircraft Trust" had under its authority the entire air traffic service including the manufacture of aeroplanes for air traffic, air survey, and sport flying, and the repairs of civil aeroplanes and airships. These two organisations maintained airframe factories in Moscow (Plant No. 1), Nizhni Novgorod (Plant No. 21), Fili (Plant No. 22), Leningrad (Plant No. 23), Taganrog (Plant No. 31), Voronezh (Plant B), and Kharkov (Plant CH). The primary aero-engine plants were located in Moscow (Plant No. 24), Rybinsk (Plant No. 26), Zaporozhe (Plant No. 29), and Perm (Plant M). Four aircraft accessory factories (Plants Nos. 20, 28, 30, and 33), in Moscow, also produced carburettors, fuel pumps, armament, propellers, skis, paints and varnishes, bolts, nuts, and so on. In addition, many smaller repair shops for both aeroplanes and aero-engines were located in such places as Leningrad, Smolensk, Kiev, and Sevastopol.

Aside from the firms directly connected to the aircraft trusts, the Soviet aeronautical industry was dependent upon other allied "trusts" and the basic commodity output. Wood, steel, aluminium, rubber, and many other less important basic commodities had to be produced in sufficient quantity and quality to supply the aircraft trusts with the necessary materials; similarly, timber, iron ore, coal, bauxite, and other such raw materials were necessary to produce wood, steel, and aluminium. The "War Aircraft Trust" had wood in sufficient quantities as it had control over certain forest regions. High quality steel had been manufactured in the Soviet Union for several years before 1932; stainless steel was produced at the Ischewoker plant while the manufacture of chrome-molybdenum steel tubing was undertaken by the Krematerski plant. Most of the aluminium, required for the aluminium alloys, still had to be imported, but an aluminium smelting plant and a water-power plant were under construction on the Dnieper River. Accessories, such as tyres and elastic cords, were being made by the "Rubber Trust" in Leningrad, and new rubber mills were being built at Yaroslavl. Magnetos, spark plugs, lighting equipment, and other electrical apparatus were produced at a new works in Moscow. The "Fine Mechanics Trust" was making all the navigating instruments, with the exception of optical equipment such as gun and bomb sights. Cameras were still being imported but the "Fine Mechanics Trust" was to begin camera production in the near future. Wireless and other radio equipment were furnished by the "Electro Trust."

Each of the individual plants and factories was given a production figure to which it was expected to adhere, and according to the plan for the Soviet aircraft industry for 1932 and subsequent years, the six military aircraft plants and four engine plants were to supply the Soviet Air Force with about 7,000 aeroplanes (4,000 single-engined and 3,000 multi-engined) by the end of 1932. Actually, only about 1,500 aircraft were produced in 1932. Even with the rapid industrial advances, the Soviet aircraft industry and basic commodity production were far behind those of the Western powers. There were still bottlenecks in aluminium and aluminium alloy output, precision tools had to be imported, aero-engine output was extremely low and many foreign engines had to be imported; oil refining processes were unable to produce a high quality fuel, and there was a decided lack of skilled technical and administrative personnel, which was reflected in the low productivity of the Soviet worker and the inefficient operation of the plants and factories. Although, by 1940, most of the foreign personnel and advisors and many unskilled Soviet workers had been replaced by newly educated Soviet technicians, designers, administrators, and other skilled personnel, the former situation continued to dominate the Soviet aircraft industry well into the Second World War.

The Second Five-Year Plan, 1933-1938

By 1935 there were about ten large airframe factories and four engine plants in the Soviet Union, producing in the neighbourhood of 4,000 airframes and 20,000 engines annually. Although most of these aircraft factories were situated in the Moscow and Ukrainian regions, the new factories were being established farther east, primarily in the Ural region. Under the second Five-Year Plan, the natural resources of the Ural mountains were to be exploited and the new industrial plants were located near the sources of raw materials*.

All of the factories producing aeronautical materials were directed by members of the Communist Party and almost all of the foreign personnel had been replaced by Soviet workers. There were still only a few capable Soviet aircraft designers, but vast educational programmes were in progress. Among the most notable Soviet designers in 1935 were A. N. Tupolev, the director of "ZAGI" and responsible for the ANT series of

*Actually this is only part of the reason for the industrial expansion to the east. By 1935, diplomatic relations with Germany were definitely hostile and in the event of an open war, all of the Soviet industrial reserve would no longer be concentrated in the western sections of the U.S.S.R. and thus would be less accessible to invaders from the West.

aircraft; Policarpov, responsible for the I-16 fighter; D. P. Grigorovitch, responsible for the I-5 single-seat fighter; and Poutilov, the designer of the Stal series of aircraft.

The technical organisation was shared by three establishments. The "ZAGI," founded in 1918, was in charge of all aircraft development work. The "ZAIM," founded in 1930, was in charge of all aero-engine development work. The "VIAM," founded in 1932, conducted and directed all the research on materials and was the contractual organisation responsible for the production of materials for the entire aircraft industry. The aircraft factories, themselves, were only producers of aircraft and maintained small and comparatively unimportant drawing offices and technical staffs.

By 1937-38, there were roughly fifteen large aircraft factories. Plant No. 1 (Moscow) had a productive capacity of about 1,500 fighter and reconnaissance craft per year; in comparison with 750 employees in 1932, about 15,000 workers were employed at this plant by 1937. Plant No. 15 (Odessa) built single-engined scouts and fighter craft. Plant No. 18 (Voronezh), earlier called Plant B, was producing twin-engined, all-metal aircraft, while Plant No. 21 (Gorki) was producing single-seat fighters such as the I-16. Plant No. 22 (Fili) was reputed to have a production capacity of from 600 to 800 all-metal bombers annually; it employed about 10,000 workers. Plant No. 23 (Leningrad) was producing amphibians and express aeroplanes. Plant No. 31 (Taganrog) and Plant No. 45 (Sevastopol) were concentrating on seaplanes and flying boats. Plant No. 39 (Moscow) built prototypes and prepared them for production while Plant No. 56 (Moscow) and Plant No. 81 (Tushino) built civil aircraft. Two additional plants, Plant No. 84 (Khimki) and Plant No. 125 (Kazan), were just being completed. By 1937-38, aircraft factories had also been built at Tomsk, Irkutsk, Khabarovsk, and Komsomolsk, all in Siberia; these factories were producing aircraft to supply the far eastern air regiments, and, for the most part, were completely independent from the western aircraft factories and allied industries.

In contrast to the results of the first Five-Year Plan, great advances in production were made in the aero-engine industry during the second Five-Year Plan. Although many types of aircraft engines, primarily of foreign design under licence, were produced at the beginning of the second Five-Year Plan, by 1937 the aero-engine industry had been revamped and only a few promising models were produced on a large scale, utilising mass production methods. Plants No. 19 (Perm), 24 (Moscow), 26 (Rybinsk), and 29 (Zaporozhe) were producing M-25 (Wright "Cyclone"), M-34, M-100 (Hispano-Suiza 12Y), and M-85 (Gnôme-Rhone 14K) engines, respectively. There were, in addition, aero-engine factories in the middle and far eastern regions of the U.S.S.R., which supplied engines for the far eastern regiments. As with the airframe factories, these aero-engine factories were independent of similar units in the western part of the country.

Under the first Five-Year Plan, all of the technical and aircraft design work was centralised under the "ZAGI." As earlier pointed out, under most of the second Five-Year Plan this work was shared by three organisations, the "ZAGI," the "ZAIM," and the "VIAM;" the aircraft factories were solely producers of aeroplanes and they carried out little technical work and almost no design work. By 1938, however, the technical and aeroplane design work was spread among a number of state aircraft factories and, eventually, it was planned to maintain an aeroplane design department in each aircraft plant. This step was taken for two major reasons—it would create opportunities for the younger engineers to demonstrate their abilities and, secondly, it would facilitate continued aircraft designs by groups that were in close contact with the practical aspect of aircraft construction. New designers and designer groups were an absolute necessity in the Soviet Union in 1938. A political "purge," during the preceding two years, had almost eliminated the Soviet Union of its then too few aeronautical engineers and designers*.

The War Years, 1940-1945

When the German military forces invaded the Soviet Union, in the late Spring of 1941, the Soviet aircraft production had reached about 1,000 aeroplanes per month and was still in a process of expansion. Although this production figure compared favourably, with respect to quantity, to the similar production figures for the western countries, the quality of the Soviet aircraft was vastly inferior to that of the German aeroplanes or, for that matter, the aeroplanes produced by the other allied nations. At this time, the Soviet aircraft industry was in the process of changing over to the production of the early MIG, the LAGG and the YAK, which eventually were to supply the Soviet Air Force with first-class fighters and fighter-bombers.

From German sources, there were about fourteen aero-engine plants in the Soviet Union at the outbreak of the war. Many of the then recently constructed aircraft plants, as well as aero-engine plants, had been located in the middle and far eastern regions of the country. Those factories that were recently constructed in the western areas of the country, where labour and transportation were readily available, were so constructed that they could be easily dismantled and moved to the east. New machinery that had been installed in the older plants in the European sector of the Soviet Union, had similarly been installed in easily transportable units, so that it could be moved in an emergency. These provisions for the evacuation of both plants and machinery proved to be extremely auspicious and this

*In 1938, A. N. Tupolev, who was the "father of aviation in the Soviet Union" and for fifteen years the chief engineer of the "ZAGI," was exiled to Siberia, along with many of his associates. Many other less fortunate designers of note, such as Kalinin, were completely annihilated. This, coupled with the hostile German relations and the knowledge of an impending war, made it imperative that new Soviet aircraft designers be obtained immediately.

system of transportation of plants and machinery was brought into wide use during the German invasion of the western regions of the Soviet Union. It can be stated, perhaps, that such provisions, along with the emphasis on heavy industry and basic commodity production of the Five-Year Plans, were, to a large degree, responsible for saving the Soviet Union from the invading German forces.

The first six months of the war had a deep, but relatively short-lived, effect on the Soviet aircraft output. The initial German ground "blitz" compelled the evacuation of more than half of the Soviet aircraft industry. Transportation facilities were heavily overloaded but priority was given to the plants and machinery that had been in the Ukraine and the Moscow and Leningrad areas, which were now being moved eastward. Some of the aircraft workers were drafted into the defending armed forces. By the end of 1941, the Soviet aircraft production had dropped to about 500 aeroplanes per month, about half of the output that had been achieved earlier. Even after the plants and machinery had been relocated in the Urals and eastern regions of the U.S.S.R., there were still many difficulties in obtaining skilled labour and raw materials. The transportation facilities were still overloaded and were desperately needed to supply the armed forces along the western front. The far eastern air regiments were depleted of many of their aircraft and Lend-Lease materials and aeroplanes from the United States and Great Britain helped to supplant the western air forces. However, grave as the situation was, by the summer of 1942 the aircraft facilities had been restored in and beyond the Urals and the aeroplane output had again risen to about 1,000 aeroplanes per month.

Both aero-engine plants and airframe plants were further developed and after 1942, the output continually increased. The aero-engine factories produced a few well-tested models, such as the M-105, AM-35, AM-38, and M-82, in quantity production; such engines were used in fighters, bombers, and reconnaissance aeroplanes. The Il-2 or "Stormovik," as it was readily known, was designed by Ilyushin in 1940, as an anti-(German) panzer weapon; this aeroplane was produced in large quantities during the early stages of the conflict and is often referred to as the Soviets' most produced aircraft during the Second World War. The MIG-3 appeared in 1941, and both the YAK-3 and LA-5 appeared in 1943; the latter became known as the "little wooden saviour of Stalingrad." In 1942, Tupolev reappeared and was restored to his former position; during the following year the Tu-2, an effective twin-engined attack bomber, was first produced.

By 1944-45, the Soviet aircraft production had been increased to between 1,500 and 2,000 aeroplanes per month. In early 1944, the movement of the earlier evacuated factories and plants back to the western regions of the U.S.S.R. had been initiated. This westward movement was almost completed by 1945, except for the fact that many units were never returned to their original sights and remained in their newer locations in and beyond the Urals.

TABLE I

MILITARY AIRCRAFT PRODUCTION IN THE SOVIET UNION, THE UNITED STATES AND GERMANY FROM 1940 TO 1945

Year	*Soviet Union**	*United States***	*Germany****
1940	9,000	6,019	10,826
1941	———	19,433	11,776
1942	12,000	47,836	15,556
1943	16,000	85,898	25,527
1944	22,000	96,318	39,807
1945	30,000	47,714	——

NOTES:

*Estimated annual production.

**Taken from a table compiled by R. Modley, appearing on page 32 in the 25th February 1952 issue of *Aviation Week*.

***Taken from a table on page 93 in the *Aircraft Division Industry Report on the Strategic Bombing of the German Aircraft Industry*, by the U.S. Strategic Bombing Survey, Aircraft Division, January 1947.

During the war, the Soviets received great stores of aid from Great Britain, the United States, and Canada. During the last three years of the war, over 15,000 aeroplanes were shipped to the Soviet Union from Great Britain and the United States. The Soviet aircraft production figures would never have been reached had it not been for the American and Canadian aluminium shipments. During the latter part of the war, about half of the aluminium that was utilised by the Soviet aircraft industry came from Canada and the United States. The Soviet aeroplanes would not have been flying in the manner they were, if it had not been for the refining plants the American and British advisors helped them build, to enable a high-grade fuel to be produced. Many American and British advisors, both technical and administrative, aided the Soviets to modernise their plants and production lines, as well as their manufacturing processes.

It would be of interest, perhaps, to compare the estimated Soviet aircraft output with the corresponding American and German output, during the war years. Table I contains the military aircraft production figures for the three countries, from 1940 till 1945. It can immediately be noted that, from 1940 to 1942, the Soviet aircraft production figures compared favourably with the German production figures. During this period, however, the Soviet aircraft industry was operating at full capacity while over-confidence prevailed in Germany—the German aircraft industry was operating at less than half of their practical production capacity. This is exemplified by the steady increase in the Soviet production figures as compared to a doubling of the German production figures in the succeeding two year period. It should also be recognised that the American and British strategic bombing was seriously restricting the German aircraft industry between 1943 and 1944; this strategic bombing was primarily responsible for the tremendous drop in the German production.

which occurred during 1944 and lasted till the end of the war.

These figures also show the effect of the tremendous industrial reserve of the United States during the Second World War. Although the United States was only producing from 65 to 70 per cent. of the aircraft that were produced in the Soviet Union in 1940, by 1944 the American production figures were over four times the Soviet production figures*. It should be remembered, however, that no matter how great an industrial reserve a country has, there is a definite period of time before such a reserve can be fully utilised; this may well be a governing factor in the future history of the world.

By the end of the Second World War, the Soviets were equipped with a relatively modern air force and they were producing between 2,000 and 3,000 aeroplanes per month. Technically, great advances had been made, in all phases of the aircraft industry, in comparison to the situation which existed in 1940-41. It may also be safely stated that the Ural industrial area had been expanded at a rate exceeding all normal expectations.

The Fourth Five-Year Plan, 1946-1950

Although American Lend-Lease aid to the U.S.S.R. ceased in the late Spring of 1945, this was more than compensated for by the mass movement of German industry† and manpower to the Soviet Union. Even before the end of the Second World War, the Soviets had begun the dismantling of the German factories and industrial plants, which were later shipped to the U.S.S.R., along with the precision machinery and other industrial material, as a partial payment of the German war reparations to the Soviet Union. The Heinkel Works at Rostock, the Dornier Works at Wismar, and the Junkers Works at Dessau, as well as almost all of the other modernised aircraft and industrial plants in the Soviet-occupied zone of Germany, supplied facilities, materials, designs under development at the end of the war, and even manpower and German "brains" to the Soviet Union in the post-war years. By 1948, virtually the whole of the Junkers Works had been re-established in the U.S.S.R.; an experimental development section of this company was located 100 miles to the north of Moscow while its main production plant was located in the Kuibyshev area.

Many of Hitler's top economists, industrial, and technical experts**, as well as tremendous numbers of skilled German aircraft workers, soon showed up working in the Soviet Union. Aside from the personnel that were recruited to work for the Soviets within the territorial boundaries of the U.S.S.R., there were countless other individuals who remained in the Soviet occupied sectors of Germany, working for the Russians. Most of these, who were technicians and scientists, continued working on the same projects they had been working on during the last stages of the war. When such men as Professor Bock, research chief of the German Air Ministry, were made available, they usually recruited a large part of their staff (of the Peenemunde Project, in the case of Professor Bock) to continue their work under the Soviets.

It would almost be sufficient to state that the Soviets obtained a sample of every German design or completed aircraft that had been produced during the latter stages of the conflict. Such aircraft as the Arado-234 (a twin-jet reconnaissance bomber), the Heinkel-162 (a jet fighter), the Henschel-132 (an experimental jet-propelled dive bomber), the Horton-9 (an experimental jet-propelled flying wing), the Junkers-287 (a four-jet bomber), the Messerschmitt-163 "Komet" (a rocket-propelled tailless intercepter-fighter monoplane), the Messerschmitt-262 (a twin-jet fighter), and the DFS-346 (a supersonic aeroplane similar to the American Bell X-1) fell into Soviet hands and provided an excellent basis for the more recent Soviet jet-propelled aircraft designs. Aside from actual completed aircraft, experimental or proven, the Soviets also obtained many of the partially completed designs which were to be the German aircraft of the future. The completed aircraft and developments in the design of jet-propelled aircraft afforded prototypes for the Soviet designers and allowed such men as Ilyushin, Lavochkin, Mikoyan, Tupolev, and Yakovlev to quickly produce such aeroplanes as the Il-19 (a four-engined high wing jet bomber), the La-15 (a twin-engined jet fighter), the MiG-15 (a single-seat swept-back wing jet fighter), the Tu-10 (a twin-engined jet attack bomber), and the Yak-23 and -25 (a twin-engined jet attack fighter and a single-seat high-speed jet-rocket fighter, respectively).

The German war booty also included the BMW-003 turbo-jet and the Jumo-004 jet engine, from which such men as Svetzov copied the Soviet M-003 and M-004 jet engines.

Of all the German acquisitions, perhaps the one

*Not only were the American production figures over four times the corresponding Soviet production figures for 1944, but the American aircraft were also much larger and heavier; at this time the Americans were concentrating on a strategic air force while the Soviets were concentrating on fighters, pursuit aircraft and attack bombers.

†Over 68 per cent. of the German aircraft production facilities, at the end of the war, were located in the Soviet-occupied territories; the Soviets obtained facilities from the Germans to produce well over 25,000 aircraft annually. Of the German airframe producing plants, the present Soviet-occupied territory includes 63 per cent. of the total German plants that produced pursuits and fighters in 1944, 91 per cent. of the total German plants that produced light and medium bombers in 1944, almost 98 per cent. of the total German plants that produced heavy bombers in 1944, 42 per cent. of the total German plants that produced transport aeroplanes in 1944, and about 73 per cent. of the total German plants that produced other miscellaneous types of aeroplanes (trainers, observation aircraft, etc.) in 1944. These figures have been obtained from the German Air Ministry and are further broken down in the U.S. Strategic Bombing Survey report (*see* footnote *** in Table I).

**Among those men that are applying their talents in the service of the U.S.S.R. are such individuals as Anton von Poller of Hitler's economic staff, Siegfried Guenther, the chief engineer at the Heinkel Works, and Dr. Baade, Junkers' chief designer, and the man responsible for some of Germany's most advanced (at that time) jet bombers.

outstanding project that was acquired by the Soviets was the entire "Peenemunde Guided Missiles Centre," on the shores of the Baltic. This organisation was the research and development centre for all the German flying bombs, guided missiles, and rockets, and it was amazingly intact at the end of the war. Here, the Soviets obtained the FZG-76 (V-1) flying bomb and the A-4 (V-2) medium range rocket, as well as all of the experimental and design data for future, more potent missiles. There were a large number of rockets that were in an unfinished state in 1945 and the Soviets lost no time in taking over this centre and continuing its work where the Germans had left off.

Although the Soviets emerged from the Second World War with a large and comparatively modern air force, they were still, technically, some four or five years behind the developments in Great Britain and the United States. The German acquisitions after the war, both in material and personnel, tended to greatly narrow this gap; it is believed that by the end of the fourth Five-Year Plan, 1950, this gap had been closed with respect to the modern jet fighters and guided missiles. Since the Soviets have always been extremely interested in rocket development, it is very likely that, at the moment, they have surpassed the American and British developments in the aerodynamic design and propulsion phases of this field; it is believed, however, that the Americans and British are still several years ahead of the Soviets in the development of control and guidance systems for rockets and other types of guided missiles.

By 1950, there were some twenty-five to thirty major airframe plants, about fifteen aero-engine plants, and some forty to fifty aircraft component factories in the U.S.S.R. There were also an indeterminate number of smaller factories and units, which produced allied equipment for these plants. Production was still controlled by the fairly efficient system of "trusts." The "Ministry of Aviation Industry" supervised a total of some 300 state factories, of various sizes. Directly under the "Ministry of Aviation Industry" were the "Central Workers Registry" and the "Academy of Aircraft Industry." The "Central Workers Registry" had complete control over the Soviet aircraft workers and allocated manpower accordingly, sometimes in a seemingly ruthless manner with severe penalties for absenteeism and tardiness. In 1947, the "Academy of Aircraft Industry" was set up for the purpose of training supervisors and foremen, as well as other minor personnel in the aircraft industry. Both of the above agencies supplied manpower for the individual trusts and the plants controlled by these trusts. Also under the "Ministry of Aviation Industry" were the various aircraft trusts—the "Scientific Motor Institute" controlled the output of powerplants; the "Rubber Trust" provided tyres, fuel cells, fittings, and other rubber equipment; the "Electro Trust" was in charge of radar and radio instruments, lights, tubes, and other electrical equipment; the "Arms Trust" was concerned with armament, armour, and ammunition; the "Airframe Trust" was, of course, concerned with the production of airframes; and, finally, the "Fine Mechanics Trust" had charge of the bomb and gun sights, aerial cameras, and other instruments. Each of these trusts had under their jurisdiction, numerous factories and plants which produced the products of their concern.

Removed from the trusts and other producing organisations under the "Ministry of Aviation Industry," this ministry also controlled what were called the "Central Aircraft Factory" and the "Prototype Farm." The "Central Aircraft Factory" built almost all the prototypes and then these aeroplanes were test-flown at the "Prototype Farms" before production was actually authorised in the designer's own plant.

The Fifth Five-Year Plan, 1951-1955

In 1953, the general organisation structure of both the "Ministry of Aviation Industry" and its sub-units was essentially unchanged from that just described as existing in 1950. Since the beginning of the third Five-Year Plan, the Soviets have been expanding their industrial reserve to the far east as well as the regions around and beyond the Urals. Before the Second World War, they had achieved two almost independent industrial and military areas; this was initiated primarily as a defence measure to ensure the Soviet Union of the ability to carry on a war on two fronts simultaneously. At the present time, the Soviet Union is divided into three, almost completely independent, military-economic units, each of which can act, if necessary, independently of the other two. The western unit extends from East Germany on the west to a line roughly following the Ural Mountains down to the Caspian Sea; the central unit extends from the Ural Mountains eastward to a line from Lake Baikal, on the south, to the Laptev Sea, on the north; and the eastern unit extends from the boundary of the central unit to the Pacific Ocean. Each of these units now has its own aircraft industries, military units, capital and consumer industries, and agricultural areas. The reasons for such a breakdown are self-evident. Aside from exploiting the natural resources of the east, if the Soviet Union becomes engaged in a conflict, it will be able to carry on a war on three fronts simultaneously, and a loss of any one area would not seriously affect the others. This process of division also serves to expand the industrial capacity of the Soviet Union over its entire 8,800,000 square-mile sphere, thereby making it far less vulnerable to strategic bombing from any opposing air forces.

Although the fourth Five-Year Plan showed a cut in military expenditures (*see* Fig. 1) and much effort was devoted to the recovery from the Nazi destruction, by 1950 the country had exceeded the pre-war industrial peaks by forty-five per cent. The Soviet Army, although smaller than at the end of the Second World War, was mechanised and had far superior fire power. The Soviet Air Force had been re-equipped with jet fighters and bombers.

An abrupt change occurred in 1951, which technically initiated the fifth Five-Year Plan; this change was probably due, in part, to the increased resistance on the Korean battleground. In 1951, direct

defence expenditures rose to 93·4 billion roubles or 21·3 per cent. of the total Soviet budget; these expenditures were further raised to 23·9 per cent. of the Soviet budget (113·8 billion roubles) in 1952. During the present Five-Year Plan, as published on 20th August 1952, the military and defence appropriations are continually increasing and the gross industrial production should be increased by about 70 per cent. by 1955 over the 1950 figure.

The estimated basic commodity output of some of the more important industrial commodities of the U.S.S.R., for the years 1940 to 1956, is presented in the Appendix. These figures are, perhaps, on the conservative side, for it is assumed that only 80 per cent. of the present Five-Year Plan goals are attained. The Plan does, however, propose large increases in the basic material production, fuel, and power, as well as in the production of non-ferrous metals*. The production of consumer goods is also scheduled to increase, but by no means as much as the capital goods†.

Estimate of Soviet Aircraft Production

Many estimates of the Soviet aircraft production figures have been advanced within the past few years. General George C. Kenney, one of the top U.S. Air Force commanders of the Second World War, said publicly in April 1948:

> "We believe that they (the Soviets) are capable of producing upwards of 40,000 aircraft a year."

General Hoyt S. Vandenberg reported, in 1952:

> "During the past five years, the Soviet Union has produced five times as many planes as has the United States."

In July 1951, *Aviation Age* estimated the Soviet aircraft production for the year 1950 to be 19,000 aircraft. The 1952 edition of *Jane's All the World's Aircraft* estimated an annual Soviet aircraft production of 22,000 aeroplanes in 1951 as compared with less than 11,000 aeroplanes produced in the United States in 1952. In the February 1953 issue of *Fortune*, the editors have made statements such as:

> "Since the end of World War II, the Soviet aircraft industry has produced about 40,000 military aircraft —about twice the estimated U.S. total during this period.
>
> "Soviet production is currently running at about 10,000 military aircraft a year—compared with a 1952 U.S. output (Air Force and Navy) of about 9,000 and a 1953 prospect of about 11,000. The Soviet aircraft industry is operating well below factory capacity, in part because of acute material shortages . . ."

Actually, numerous estimates of the Soviet aircraft production have been advanced during the past few years and these estimates have ranged from 10,000 aircraft per year to an annual production as high as 100,000 aircraft.

Since such a range of estimates of Soviet aircraft production figures has been advanced by various individuals and groups, it seems appropriate to analyse the validity of such estimates. Inasmuch as the aircraft production industry is so closely dependent upon the aluminium industry*, as well as the other basic commodity industries, it would indeed be simple to obtain an upper limit to the Soviet aircraft production figures. In addition, from a technical knowledge of the aircraft industry, from comparative percentage production figures in Great Britain, the United States, and Germany during the Second World War, and from some tactical knowledge of the utilisation of modern air power, it is even possible to make a moderate estimate of the Soviet aircraft production figures for some of the past and immediate future years. Such an estimate will be presented but it is well to remember that this estimate is valid only in so far as its basic assumptions are valid.

The primary assumption in the following estimate is two-fold. The estimate is essentially based upon the aluminium production in the Soviet Union for the various years, from 1948 to 1956, and the assumption that 75 per cent. of the total tonnage of aluminium produced is allocated to the aircraft industry for the production of aircraft and their components. The aluminium production figures, for the Soviet Union during the years 1948 to 1950, have been obtained, as estimates, from the Commodity Research Bureau. The aluminium production figures for the years 1951 to 1955 are based on the assumption that the goals of the fifth Five-Year Plan are 80 per cent. fulfilled and that the yearly percentage increase in aluminium production over

*Coal production is scheduled to be increased by 43 per cent. during the five-year period, crude petroleum is to show an increase of 85 per cent., electric power is to be increased by 80 per cent., pig iron by 76 per cent., crude steel by 62 per cent., rolled steel by 64 per cent., and cement by 120 per cent. During the quinquennium, aluminium output is scheduled to be increased by 160 per cent., copper should be increased by 90 per cent., lead by 170 per cent., nickel by 53 per cent., tin by 80 per cent. and zinc by 150 per cent. The output of coking coal should be raised by 50 per cent. and the building and metal processing industries should be doubled. The capital investment in industry should reach twice the level of the preceding five-year period. In the engineering and chemical industries, if the 1950 production index is 100, the 1955 index targets are 230 for steam-turbines, 780 for hydro-turbines, 185 for metallurgical equipment, 350 for oil equipment, 260 for large machine tools, 120 for motor cars, 119 for tractors, 179 for caustic soda, 188 for mineral fertilisers, and 182 for synthetic rubber.

†Cotton materials are to be increased by 61 per cent., woollen materials by 54 per cent., leather footwear by 55 per cent., sugar by 78 per cent., animal fats by 72 per cent. and vegetable fats by 77 per cent.

*At the end of the Second World War, the three bottlenecks in the Soviet aircraft industry were aluminium production, aero-engine production and precision machinery production. The requirements for precision machinery were adequately fulfilled by the acquisitions from the German factories and plants. Similarly, the engine production has been greatly advanced by both German material and manpower. It remains, therefore, that the aluminium production is, in all probability, the outstanding present bottleneck in the Soviet aircraft industry.

TABLE II

AVERAGE WEIGHT OF ALUMINIUM PER AEROPLANE FOR VARIOUS TYPES OF AEROPLANES

Type of Craft	*Average Weight of Aluminium per Aeroplane in Metric Tons*
Pursuits and Fighter-Bombers	4·5
Light and Medium Bombers	10·0
Heavy Bombers	25·0
Transport Aeroplanes	11·4

the five year period is a constant. The aluminium production figure for the year 1956 was obtained by assuming the same yearly percentage increase in aluminium production that was utilised during the preceding five years. The assumption that the fifth Five-Year Plan is 80 per cent. fulfilled is, perhaps slightly conservative. The assumption that 75 per cent. of the total tonnage of aluminium produced is utilised by the aircraft industry is the most questionable of the assumptions; although this figure would be less than 20 per cent. in such countries as the United States, in the Soviet Union it is necessarily high due to the expressed objectives of the Soviet leaders, by which many consumer goods have to be sacrificed in order to build "priority" defence items.

Another assumption of importance is that the expressed objective of the Soviet Air Force, from the end of the Second World War till 1952, has been to build up an impenetrable defensive air power, capable of defending the Soviet and allied territories from any possible outside aggression. Having substantially fulfilled this goal, the guiding powers of the Soviet Air Force would then increase the relative emphasis of the striking power or strategic arm of the Air Force. Translated into practical terminology, this means that the major emphasis of the Soviet aircraft producing industry, from the end of the Second World War till (approximately) 1952, would be on fighter and pursuit aeroplanes and attack bombers. In 1953, the percentage of medium and heavy bombers that will be produced will increase slightly at the expense of the percentage of fighters and pursuit aeroplanes. This relative increase in the percentage of bombers may be further increased in 1955. Although the justification of this assumption could be discussed at length, as is also true of the other assumptions, such a discussion would rightfully require an additional text; it is sufficient to state that this has been a carefully considered assumption and is reflected by the opinion of the author as well as by the general philosophy of the Soviets.

Two other minor assumptions are utilised in the estimate of the Soviet aircraft production. The average weights of the various aeroplanes of each type—*e.g.* pursuit and fighter-bomber, light and medium bomber, heavy bomber, transport aircraft—have been obtained by literally averaging the weights of the recent Soviet aircraft. The additional assumption was made that 70 per cent. of the total weight of aeroplanes weighing ten tons or less is aluminium and 60 per cent. of the total weight of all aeroplanes weighing over ten tons is reflected in aluminium.

TABLE III

PERCENTAGE DISTRIBUTION OF TOTAL AEROPLANES PRODUCED IN THE U.S.S.R.

Type of Craft	*Percentage of Total Aeroplanes Produced*		
	1948-1952	1953-1954	1955-1956
Pursuits and Fighter-Bombers	60	56	54
Light and Medium Bombers	25	27	28
Heavy Bombers	7	9	10
Transport Aeroplanes	8	8	8

In estimating the aircraft production in the Soviet Union, let us divide the total number of aircraft into four distinct groups—pursuits and fighter-bombers, light and medium bombers, heavy bombers, and transports; it will be assumed that the trainer and observation aircraft, as well as other auxiliary types are either included in these groups, or are made up of obsolete aircraft from these groups, or are made from other materials than aluminium (such auxiliary aircraft not included in the above groups, in one form or another, comprise an extremely small percentage of the first-line air power). By averaging the weights of recent Soviet aeroplanes belonging to each group and assuming that 70 per cent. of the total weight is aluminium for all aircraft weighing ten tons or less and 60 per cent. for the aircraft weighing over ten tons, the average weight of aluminium per aeroplane is shown in Table II. For the years 1948 to 1956, assume that the distribution of the aeroplanes produced by the Soviets is that shown* in Table III. As can be immediately noted, through 1952, the major emphasis has been centred upon the production of fighters and defensive tactical aircraft. The relative percentages, after 1952, reflect an appreciable increase in emphasis on the strategic aircraft.

The above figures, knowledge of the total anticipated aluminium output in the Soviet Union, and the assumption that 75 per cent. of the total tonnage of aluminium produced is utilised in the aircraft industry enable one to estimate the Soviet aircraft production from 1948 till 1956. Let A be the total metric tons of aluminium produced in the Soviet Union during any of the mentioned years, while x is the total number of aircraft

*This distribution reflects the earlier stated objectives of the Soviet Air Force. It has been obtained from a comparison with the similar figures for the other countries, during the past ten years. For purposes of comparison, the following table is presented for the German war-time production (taken from the *Aircraft Division Industry Report on the Strategic Bombing of the German Aircraft Industry*, by the U.S. Strategic Bombing Survey, Aircraft Division, January 1947):

Type of Craft	*Percentage of Total Aeroplanes Produced*					
	1939	1940	1941	1942	1943	1944
Pursuits and Fighter-Bombers	22·7	28·7	31·7	33·5	46·0	72·7
Light and Medium Bombers	35·0	36·5	36·4	40·5	31·7	15·0
Heavy Bombers	0·1	0·4	0·5	1·6	1·9	1·3
Transport Aeroplanes	11·4	7·1	8·2	8·1	8·0	2·5
Others	30·8	27·3	23·2	20·3	12·4	8·5

produced during the same year. Then, for the years 1948 to 1952, the following equation may be obtained by equating 75 per cent. of the total weight of aluminium produced to the total tonnage of aluminium utilised by the Soviet aircraft industry:

$$\text{I.}\quad 0{\cdot}75A = 0{\cdot}60\,(4{\cdot}5)\,x + 0{\cdot}25\,(10{\cdot}0)\,x + 0{\cdot}07\,(25{\cdot}0)\,x + 0{\cdot}08\,(11{\cdot}4)\,x$$

Similarly, for the years 1953 and 1954:

$$\text{II.}\quad 0{\cdot}75A = 0{\cdot}56\,(4{\cdot}5)\,x + 0{\cdot}27\,(10{\cdot}0)\,x + 0{\cdot}09\,(25{\cdot}0)\,x + 0{\cdot}08\,(11{\cdot}4)\,x$$

And, for the years 1955 and 1956:

$$\text{III.}\quad 0{\cdot}75A = 0{\cdot}54\,(4{\cdot}5)\,x + 0{\cdot}28\,(10{\cdot}0)\,x + 0{\cdot}10\,(25{\cdot}0)\,x + 0{\cdot}08\,(11{\cdot}4)\,x$$

These equations can then be solved for x, the total number of aircraft produced in a given year, and in terms of A, the total aluminium produced during the same year. If the equation, $0{\cdot}75A = 1{\cdot}00(4{\cdot}5)x$, is similarly solved for x, an estimate of the maximum possible total aircraft output is obtained—that is, the maximum number of aeroplanes the Soviets could produce if they produced only fighters and pursuit aircraft, the lightest of combat aeroplanes. The aluminium production figures may be obtained from the table of "Estimated Basic Commodity Output of the U.S.S.R.—1940-1956," which appears in the Appendix. Thus, the following table of the "Estimated Aluminium and Aircraft Production of the U.S.S.R.—1948-1956" may be obtained:

TABLE IV

ESTIMATED ALUMINIUM AND AIRCRAFT PRODUCTION OF THE U.S.S.R. 1948-1956

Year	*Production of Aluminium (metric tons)*	*Estimate of the Total Aircraft Output*	*Estimate of the Maximum Total Aircraft Output (all fighters)*
1948	140,000	13,350	23,300
1949	165,000	15,730	27,500
1950	190,000	18,100	31,700
1951	224,000	21,400	37,400
1952	264,000	25,200	44,000
1953	311,000	27,800	51,800
1954	367,000	32,800	61,100
1955	433,000	37,600	72,200
1956	510,000	44,300	85,200

BREAKDOWN OF THE ESTIMATE OF THE TOTAL AIRCRAFT OUTPUT

Year	*Pursuits and Fighter-Bombers*	*Medium Light and Bombers*	*Heavy Bombers*	*Transport Aeroplanes*
1948	8,010	3,340	930	1,070
1949	9,440	3,930	1,100	1,260
1950	10,860	4,520	1,270	1,450
1951	12,840	5,350	1,500	1,710
1952	15,120	6,300	1,760	2,020
1953	15,580	7,500	2,500	2,220
1954	18,370	8,850	2,950	2,630
1955	20,310	10,520	3,760	3,010
1956	23,930	12,400	4,430	3,540

TABLE V

U.S. AIRCRAFT PRODUCTION HISTORY 1940-1952

Year	*Military Aeroplanes*	*Civil Aeroplanes*	*Total Aeroplanes*
1940	6,019	6,785	12,804
1941	19,433	6,844	26,277
1942	47,836	0	47,836
1943	85,898	0	85,898
1944	96,318	0	96,318
1945	47,714	2,047	49,761
1946	1,669	35,001	36,670
1947	2,100*	15,617	17,717*
1948	2,300*	7,302	9,600*
1949	2,500*	3,600*	6,100*
1950	3,000*	3,520*	6,520*
1951	4,500*	2,570*	7,070*
1952	9,000*	4,000*	13,000*

*Estimated.

It can be immediately seen that even if the Soviets were producing only fighter and pursuit aeroplanes, estimates of from 50,000 to 100,000 aeroplanes annually are erroneous and have no basis. On the other hand, the estimated production of 19,000 aircraft in 1950 by *Aviation Age* seems to be in close agreement with the figure of 18,100 aircraft, for the same year, that has been obtained in this estimate. Similarly, the production figure of 22,000 aircraft for 1951, advanced by *Jane's All the World's Aircraft*, seems to be in agreement with the figure of 21,400 that was obtained in this estimate.

There also appears to be a substantial basis for the statement advanced by General Vandenberg. General Vandenberg claimed that during the past five years, the Soviet Union has produced five times as many aeroplanes as has the United States. Table V contains a table of the U.S. aircraft production figures, compiled (except for the 1952 figures) by R. Modley and published in the 25th February 1952 issue of *Aviation Week*. From 1948 to 1952, the United States produced about 21,300 military aircraft; the comparable figure from our estimate, for the Soviet Union is 93,780. These figures appear to substantiate General Vandenberg's statement quite readily.

Many intelligence reports concerning the Soviet aeronautical progress are submitted to the authorities in the U.S.A. and other Western countries, but even these reports show evidence of conflicting statistics. Actually, the validity of the figures advanced herein, or any other figures of this nature, can only be verified by the Soviets themselves. The important factor is that the Soviets are continually expanding and raising their production levels and at the moment they appear to be producing more than twice the number of aircraft that are being produced in the United States. By the end of the fifth Five-Year Plan, the annual Soviet aircraft production should reach a level of 37,600 aeroplanes and, by 1956, the Soviets could be producing about 44,000 aeroplanes per year, many of which would be classified as strategic weapons. As can be seen in Table V, even with its vast industrial reserve, it took the United States almost two years to reach an equivalent rate of production during the Second World War.

APPENDIX

ESTIMATED BASIC COMMODITY OUTPUT OF THE U.S.S.R. 1940-1956

No.	*Commodity*	1940*	1941*	1942*	1943*	1944*	1945*	1946*	1947*	1948*	1949*	1950*	1951**	1952**	1953**	1954**	1955**	1956***
1.	Aluminium (in thousands of metric tons)	59·94	66·4	55·0	62·34	71·0	86·31	105·0	120·0	140·0	165·0	190·0	224·0	264·0	311·0	367·0	433·0	510·0
2.	Chromite (in thousands of metric tons)			400·0	325·0	300·0	300·0	300·0	500·0	600·0	350·0	500·0						
3.	Coal (in millions of metric tons)	165·0	147·0	90·0	131·0	118·0	146·0	161·0	175·0	209·0	236·0	264·0	280·0	298·0	317·0	337·0	358·0	380·0
4.	Coke (in millions of metric tons)	16·5		7·3	8·3	11·0	13·0	14·5	17·0	20·0	24·0	27·0	29·0	31·0	33·0	35·5	38·0	41·0
5.	Copper (in thousands of metric tons)	157·0	160·0	135·0	130·0	135·0	140·0	150·0	165·0	180·0	200·0	218·0	243·0	271·0	302·0	337·0	376·0	420·0
6.	Iron Ore (in millions of metric tons)	27·5	22·7		14·0	16·0	18·0	21·0	24·0									
7.	Pig Iron (in millions of metric tons)	15·2	11·45	4·28	5·5	7·21	8·73	10·0	11·2	14·1	16·7	19·5	21·5	23·6	26·0	28·5	31·4	34·5
8.	Steel (in millions of metric tons)	18·2	15·0	11·1	12·1	14·0	18·0	18·1	19·9	20·1	20·6	24·5	26·6	28·8	31·3	34·0	36·9	40·1
9.	Lead (in thousands of metric tons)	75·0	90·0	80·0	50·0	45·0	40·0	48·0	63·0	75·0	90·0	104·0	123·0	147·0	174·0	206·0	245·0	291·0
10.	Magnesium (in thousands of metric tons)	1·5	4·0	5·0	5·0	5·0	2·17	3·0	4·0	5·0	5·0							
11.	Manganese (in millions of metric tons)	2·8	2·39	1·82	1·0	0·46	2·25	1·7	1·8	1·8	1·5	2·0						
12.	Nickel (in thousands of metric tons)	8·65	13·6		11·16	13·0	13·4	20·0	25·0	25·0	25·0	25·0	27·0	29·0	31·0	33·0	36·0	38·5
13.	Petroleum (in millions of barrels)	218·6	238·15	227·47	200·75	275·0	148·95	157·67	187·46	218·0	237·7	266·2	296·0	329·0	365·0	405·0	450·0	500·0
14.	Zinc (in thousands of metric tons)	85·0				84·0	89·0	90·0	106·0	110·0	110·0	129·0	151·0	177·0	207·0	243·0	285·0	334·0

NOTES:

*Data taken from the Commodity Research Bureau estimates.

**Data estimated from the 1950 figures plus the assumption that the Fifth Five-Year Plan is 80 per cent. fulfilled.

***Estimation is based on the extrapolation of the Fifth Five-Year Plan figures.

TECHNICAL NOTES

Contributions to this Section of the JOURNAL *will be eligible for Journal Premium Awards and will normally be published within two months of being received.*

Some Unusual Aspects of Turbulence

By

CAPTAIN F. D. BETHWAITE, D.F.C.

(This paper was given before the Auckland Branch of the New Zealand Division of the Society on 29th October 1953. It is based largely on the author's experience and, although his approach to the subject of turbulence is perhaps unusual, the paper is interesting and the opinions expressed are thought to be worth considering—EDITOR.)

THE first idea about unsteady motion in the atmosphere seems to have been the vague notion of "Air Pockets"—areas or gaps, wherein the air just does not generate lift, and an aeroplane falls in consequence. It is astonishing how widely this view is still held.

Those engaged in aviation soon learned that unsteady flight was due to turbulence and two sorts, friction and orographical, were quickly recognised. Friction turbulence, which affects only the lower layers of the atmosphere, can best be observed by watching a soaring sea-bird in a breeze. The friction of the water surface creates a velocity gradient or variation of wind-speed with altitude, and the bird will probably be diving down-wind, and climbing into wind, with no effort—from the bird's point of view a fair approach to perpetual motion. However, the air is really a mass of tiny eddies where friction is at work and close observation of the bird's wings and feathers will show them to be constantly in motion as they yield to each puff.

Orographical turbulence, the motion caused by wind blowing over hills and the like, can also be best observed by watching its effect on water. Evans Bay in Wellington yields some classic examples. The mechanism seems to be that a long horizontal vortex forms along the lee side of a ridge and then, breaking away, descends until it impinges on the surface and causes high-speed gusts to fan out from the point of impact at the moment. These gusts are characteristically abrupt in rising to their maximum velocity and the velocity difference between gusts and lulls is frequently more than 50 ft./sec. The extreme example of orographical turbulence is perhaps the generation of huge standing waves, sometimes reaching clear to the stratosphere behind great mountain ranges, and much beloved of glider pilots because of their promise of sustained flight.

Understanding of a third form, convention turbulence, had necessarily to await the development of flight instruments which, in turn, permitted cloud flying and cloud penetrations by gliders appear to have furnished much of the early material for conjecture. Later, as aircraft came to penetrate all kinds of clouds, knowledge of what can happen within a convection area has been built up. As a result of this knowledge we now insist that our aircraft be built strongly enough to withstand at least the greatest stresses expected to be imposed.

I.C.A.O. REQUIREMENTS

What is probably the most thorough investigation of convection turbulence ever to be undertaken was completed by the United States Army Air Corps shortly after the 1939-45 War, and the findings have been published. Shortly thereafter *V-g* recorders, to indicate the magnitude and frequency of vertical accelerations and the indicated air speeds at which they occurred, were carried in civil aircraft the world over for a period of several years. The knowledge thus gained of loads resulting from turbulence has been used to frame the airworthiness requirements recommended by I.C.A.O. for adoption by all member states.

I would like to draw attention to two particular points in these requirements. First, the Gust *V-n* diagram requires that the structure be able to withstand, below 25,000 ft., vertical gusts of ± 66 ft./sec. at the associated stall speed; ± 50 ft./sec. at the design cruising speed; and ± 25 ft./sec. at the design diving speed. Note particularly the relatively low strength required to meet down-gusts, and that the manoeuvring strengths required do not much modify this picture. Secondly, that strength to withstand lateral gusts of 50 ft./sec. is all that is required.

To sum up the record so far, it is interesting to quote figures. The maximum vertical gust recorded during the U.S.A.A.F. Thunderstorm Project was of the order of 43 ft./sec. (sharp edge equivalent) during a sequence in which loads alternated from $+1{\cdot}5$ to $-1{\cdot}15$ to $+2{\cdot}0$ to $-1{\cdot}45g$'s over 5 seconds. The most severe load recorded by New Zealand aircraft fitted with *V-g* recorders was about $+2{\cdot}0g$. I.C.A.O. require strength to withstand a ± 50 ft./sec. gust at cruising speed, and in any case strength to withstand loads of not less than $+2{\cdot}5g$ and $-1{\cdot}0g$. Agreement between experience and requirements in the vertical gust sense is seen to be close. Concerning horizontal gusts and lateral control there is

not one word in the Thunderstorm Project's report, and I.C.A.O. considers 50 ft./sec. to be a safe coverage.

PERSONAL EXPERIENCES

A few years ago, while flying a Sandringham (converted Sunderland Mk. 5) on the Auckland-Sydney route, a patch of violent turbulence was encountered while traversing an occlusion at about 7,000 ft. One particular sequence impressed me—we were holding an average speed in the 120-140 kts.* region, when speed fell off to below 80 kts. and held steady for about two seconds, then abruptly rose to 165 kts., again for about two seconds, before reverting to an average 120-140 kts. again. The transition from 80 to 165 was at a rate which caused the air speed indicator to move faster than the eye could follow. The implication here is that the aircraft passed through a horizontal gust, approximately sharp-edged in character, of some 144 ft./sec. velocity.

All speeds quoted here are indicated and are uncorrected for altitude—they are therefore approximately equal to the equivalent air speeds used throughout the I.C.A.O. requirements.

In the early winter of 1952 an almost identical sequence was repeated. On the climb out of Sydney in a Solent Mk. 4, at about 10,000 ft., an unexpected upper front buried in thick nimbo-stratus caused a few unpleasant minutes. During one sequence on this occasion speed fell from the 150 kts. I.A.S. region to 98 kts. for about two seconds, then rose to 194 kts. for about two seconds, then fell to 150 kts. or thereabouts again. I knew what to expect this time and noted particularly the lowest and highest speeds, and tried to estimate the time to change; but again the air speed indicator needle moved faster than the eye could follow. The implication on this occasion was that of a horizontal gust of 162 ft./sec.

I know that random pressure variations within the cloud will cause erroneous readings of the air speed indicator, but all other indications such as noise, feel and lateral acceleration, left no doubt whatever that very large changes in relative speed had taken place very abruptly, and the values indicated seemed to me to compel attention. Also, a further corroboration occurred in a different manner. Both gusts so far referred to were from a direction more longitudinal than lateral.

A minute or so after encountering the second gust referred to, another gust struck the aircraft from the port side with extreme suddenness. Its effect was to bang the rudder hard against its stops and for about two seconds thereafter the combined efforts of two of us were unable to budge it. After this it streamed central again, and it was clear that our efforts had been small compared with the forces involved.

It is interesting to try to evaluate the velocity of this gust. Rudder deflection on the Solent Mk. 4 is of the order of 15 degrees, and the balance forces are such that one man can just apply full deflection at about 115-120 kts. I.A.S. and there is no tendency to lock. It would thus be expected that two men could just deflect

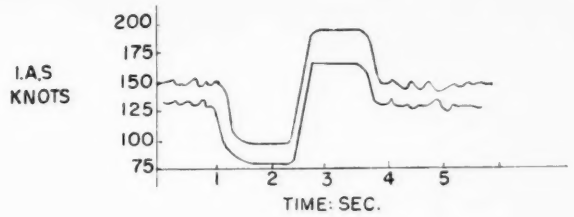

FIGURE 1.

it at 1·4 times this speed, namely 160-165 kts., and I.A.S. at the time was about this figure, perhaps nearer 155. Now, on the most empirical reasoning it would require a 30 degree cross-wind to hold the rudder deflected just balanced against the same force, and thereafter all the corrections are upwards. Specifically, the rudder was not just balanced—it was held over very firmly indeed. Also, from aerodynamic reasoning it will require a greater relative cross-wind value to hold a control fully deflected than to deflect it, due to pressure distributions around a deflected control surface. Thus a starting figure of 92 ft./sec. must be considerably increased because of the two factors mentioned, and I suggest a value of, say, 120-130 ft./sec. as the lateral component of the gust in question.

I have described three horizontal gusts of very large velocities indeed which it has been my misfortune to encounter over a period of about five years. Some other pilots whom I have questioned also recall the same sort of experiences, and usually they are associated with the same patterns of time sequences. As their recollections of limiting velocities and rates of change were not precise, I have not presented any here. Suffice it to say that very large horizontal gusts are known to occur rather infrequently in sub-tropical and warm temperate latitudes.

Some of the effects of such large gusts on the aircraft itself and the manner in which it is controlled will be discussed later, but before doing so it will be profitable to inquire what is the structure of these phenomena. How can relatively sharp-edged discontinuities of approaching 100 m.p.h. velocities be reasoned to exist in areas where the maximum wind velocities are very much less?

Consider the time structure of all the cases noted. The two gusts from ahead are shown graphically on a time-velocity basis in Fig. 1.

Film records show that the sequence of four vertical gusts which included the worst case recorded in the Thunderstorm Project's report, was one of five seconds.

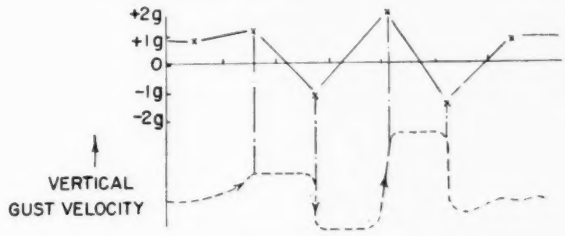

FIGURE 2.

*One knot = one nautical mile = 1·1515 miles per hour.

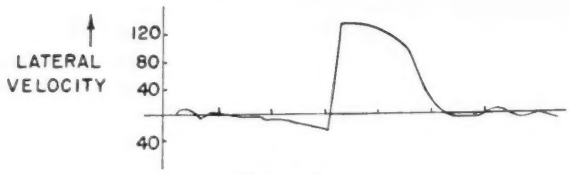

FIGURE 3.

involving four changes in *g*-loading. From this can be inferred a velocity structure such as is indicated in Fig. 2.

The lateral case is rather different and it is suggested that it would be as shown in Fig. 3. As already mentioned, other pilots have remarked this curiously consistent four or five second sequence which is associated with these enormous gusts.

WATERSPOUTS

I regard this time structure as the key. These repeated large velocity sequences over short but relatively constant periods were beginning to admit of only one explanation. Then recently I was granted by chance a grandstand view of what I am now convinced is the answer. I have seen many waterspouts, but seldom under circumstances in which I could view them at leisure and in detail. One formed in the Rangitoto Channel out towards TiriTiri, and moved up channel for about twelve minutes before it ran ashore and caused loss of life. During this time I was able to observe it continuously through powerful glasses. Its behaviour was this: for several minutes one "spout" remained active, and then decayed slowly until only a short "trunk" showed beneath the cloud. By this time a second "feeler" was forming in the immediate vicinity of the decaying vortex, rotating slowly around it, and gradually this new "feeler" became a dominant fully-developed water-spout with a life of, perhaps, five minutes. A second decay and regeneration were hidden in detail by rain curtains. However, my suspicion had been confirmed. The pair of adjacent vortices is a system which can and does develop in areas of intense liberation of energy, and any area which yields isolated waterspouts may, it seems, yield adjacent pairs closely spaced.

Now, if waterspouts can occur up to the cloud base at, say, 1,000 ft., then they can, in fact they must, continue into the cloud up to far higher altitudes. I can find no work of reference which yields any information as to the likely structure of that part of a tornado, or waterspout, which is buried in the cloud and therefore invisible. However, as such phenomena invariably grow downwards from the cloud base, therefore the circulation must already exist within the cloud. Logically it would seem reasonable to believe that the upper limit of the intense circulation involved would be most likely to lie at that altitude at which the liberation of energy is greatest. Many works on meteorology suggest that this altitude is somewhere near the freezing level, and the conclusions of the Thunderstorm Project that the 11,000 to 20,000 ft. band was more turbulent on the whole than the 6,000 ft. level tend to support theory in this respect. The upper limit may depend on the temperature, and may be very high indeed in a warm atmosphere. Therefore I suggest that a "waterspout" is not confined to just that vortex which we can see below cloud, but is a continuous vortex growing downwards from, perhaps, the freezing level within the thunderstorm wherein it generates. It may or may not grow downwards far enough to become visible below cloud base, or in the final stage reach right to the surface. At any point within the influence of such a vortex, horizontal velocities can be expected to be large in comparison with vertical velocities, and these horizontal velocities may be very great. This conflicts directly with orthodox theory which considers turbulence to result from random horizontal vortices, and thus both vertical and horizontal gusts to be equal in magnitude.

Consider one further point. Waterspouts, wherever they may occur, are of a remarkably uniform size. All works of reference quote, "A few hundred yards," as their diameter, and this size and a logical spacing between adjacent pairs such as I have observed, is exactly such as would cause an aircraft travelling at 150-200 kts. to experience the time-velocity sequences which have been so remarkable a feature of the heavy gusts referred to so far.

The explanation, I am now convinced, is that on the two occasions during which speed has fallen, risen, and then settled to its original value, the aircraft has passed between the two vortices of a pair, as shown in Fig. 4.

On the occasion of the exceptional cross-wind gust, I suggest that the track has been through a single vortex, as indicated in Fig. 5. The progressively increasing wind from starboard would not be noticed in the general turbulence, but the reversal as the aircraft crossed the centre was certainly apparent.

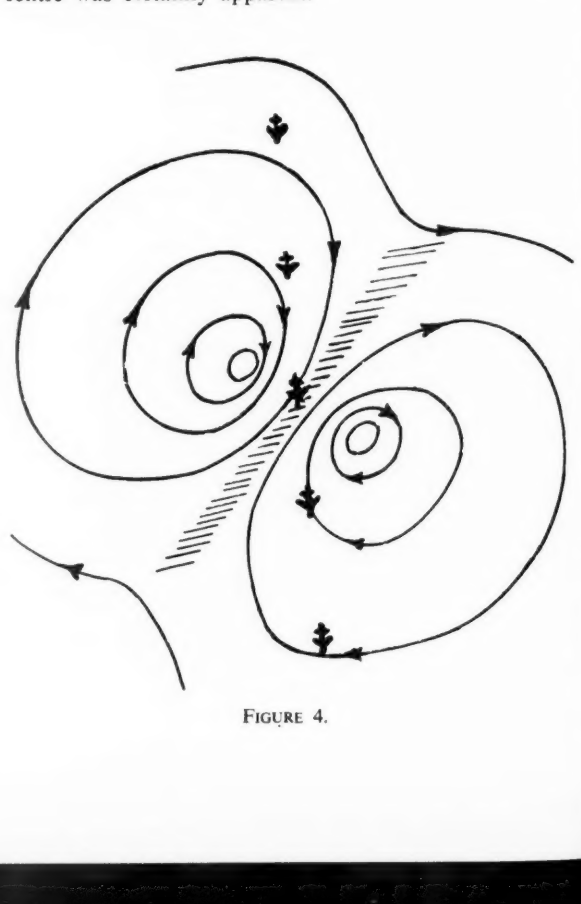

FIGURE 4.

Short flying boats, in which these gusts were experienced, have a characteristically large and effective forward fin area, and little dihedral angle. A cross-wind gust thus tends to displace them sideways bodily, with little roll or yaw, and in the cases quoted there has, fortunately, been little displacement in roll. This response is in contrast with the majority of aircraft, for with little effective vertical area forward, and large dihedral, they tend to roll rapidly under the influence of a cross wind.

Dr. Barnett, Director of New Zealand Meteorological Services, has been kind enough to discuss the subject of waterspouts with me, and in the absence of other references to their detailed structure, with his permission, his views on the subject are quoted.

He supported the argument. I had been worried by the point that a theoretical velocity distribution through a pair of adjacent vortices rotating in the same direction, may tend to give a smooth change of velocity between them. He suggested that this was not necessarily so at all; in a case such as the one under consideration, that several factors could yield a definite "front" between the centres, with high opposing velocities maintaining themselves right up to this boundary from each side.

He saw every reason to presume the existence of a waterspout's circulation to at least the freezing level.

He suggested that the vortices are of so critical a nature that they could be presumed to lie in any sense within the cloud—in other words they may not necessarily be vertical, nor must their rotation be always in the horizontal plane. This latter observation is particularly interesting when it is considered together with the vertical accelerations in the Thunderstorm Project gusts mentioned. Perhaps here the aircraft was affected by a pair of vortices the rotation of which was inclined sufficiently to yield severe vertical loadings as well as horizontal gust.

I am indebted to Dr. Barnett for his views and helpful suggestions, which greatly strengthened my own convictions.

HORIZONTAL GUSTS

To recapitulate for a moment—so far I have tried to introduce the idea of turbulence along orthodox lines, with particular reference to the fact that horizontal gusts appear to be dealt with rather summarily by the "50 ft./sec. lateral gust at cruising speed" requirement. (Another requirement that a 25 ft./sec. gust from ahead must be stressed for in the flap-down configuration has no bearing here.)

I then quoted examples wherein gusts of very high velocities have struck an aircraft with extreme suddenness from directions both ahead and abeam. In developing this argument further, I have offered what I hope has been an interesting hypothesis as to how the existence of these gusts may be explained, and I have quoted Dr. Barnett's views in support of my own.

What follows is simply an analysis of various aspects in the light of knowledge that high velocity horizontal gusts are known to exist. If these gusts do exist in a form which approximates being sharp-edged, then I

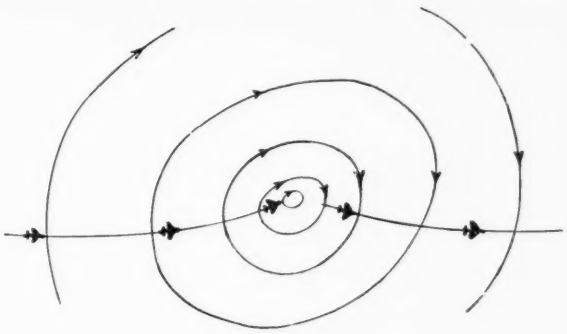

FIGURE 5.

believe that what follows is true. The question of how the gusts come into existence—of whether my hypothesis is right or wrong—does not affect the arguments.

HORIZONTAL GUSTS AS THEY AFFECT AIRFRAME STRESSES

The first question that arises is, quite obviously, that horizontal gusts have been encountered in flight of greater strength, apparently, than those which have been anticipated in design requirements. To quote the worst case, a velocity difference of 162 ft./sec. was measured on the A.S.I. over a period which I am certain did not exceed one quarter of a second.

I realise that a detailed analysis of the various errors, coupled with the maximum lift coefficient obtainable across the fin at the air speeds involved, would equate this observation to a "sharp-edge" gust of somewhat lower velocity. Nevertheless, a very large stress indeed is quite clearly under consideration and the suggestion that an aircraft may meet, in flight, stresses far beyond those designed against, cannot be other than disturbing.

HORIZONTAL GUSTS AS THEY AFFECT CONTROL SYSTEM STRESS

The next question which exercises me is the strength of the rudder control system. In one case quoted, the rudder was banged—and I mean banged—against its stops and there it stayed despite the concerted effort of two men until such time as the aircraft had accelerated laterally sufficiently to alleviate the cross-wind gust. In this case the concerted effort probably amounted to some 600 lb. or more on the foot pedals and from the feel of the control it was a long way from being enough to shift the rudder. Perhaps 1,000 lb. would have moved it—I doubt if the effort required for equilibrium would have been less.

I am well aware that control systems are customarily factored very highly and as a result are strong. This example—admittedly an extreme one—is offered to suggest that anything less than very great strength could well be disastrous.

HORIZONTAL GUSTS AS THEY AFFECT STABILITY

This may seem a rather curious aspect of flight through turbulence—stability—but flying technique will be considered later and some of my remarks therein will refer to stability.

Consider longitudinal trim. It is required, quite naturally, that a push on the control column shall be needed to maintain any speed higher than the trimmed speed, and a pull to maintain any lower speed. This is normal for any aircraft. Now if an aircraft trimmed for some certain speed encounters horizontal turbulence such that gusts from ahead and astern are encountered alternately, what will happen to trim? I believe that a normal aircraft will trim to some lower average speed, because the nose-up forces during the periods of increased air speed will be greater than the nose-down forces during the periods of decreased air speed. This checks with handling experience in practice; also this action can be observed very clearly in the behaviour of a model which is designed to take advantage of this property. The point that I wish to make at the moment is that in theory a definite average one-way force on the control column will be necessary in turbulence in order to maintain an average I.A.S. which has already been trimmed for in calmer air.

Consider an aircraft in a cross wind. Normal stability dictates that the nose will swing into wind, and that the windward wing will rise. This means that, in a cross-wind gust, the normal action of an aircraft will be to yaw towards the gust at the same time as it rolls away from it. In passing, I would like to point out that the lateral stability of an aircraft with sweepback will rise with increase of coefficient of lift. Thus a swept-wing aircraft which encounters a sustained lateral gust, will tend to roll away from it, slowly perhaps at first but with increased rate of roll as the underside of the wing becomes exposed to the gust and the coefficient of lift rises in consequence. Practical experience is that, while displacements in pitch and yaw tend to remain small, really large displacements in roll are commonplace in severe turbulence, regardless of the type of aircraft.

HORIZONTAL GUSTS AS THEY AFFECT PILOTING TECHNIQUE

The first point I would like to make is that the basic nature of instrument flight presupposes that while the pilot must ignore acceleration forces and believe his instruments instead, nevertheless the feel of the controls themselves remains unchanged from that of normal visual flight. In other words, although a pilot may on occasion get "the leans," and be forced wilfully to reject his own sensations of attitude while he interprets his instrument indications and flies accordingly; yet he has all unconsciously the reassurance that the feel of his controls is confirming his conscious labours in the art of instrument flight, and as this feel remains constant at all times *except during heavy turbulence* he cannot in fact do otherwise than accept its reassurance. This point is so obvious that it tends to be overlooked.

Now consider flight on instruments through an area wherein a large measure of horizontal gust and turbulence is encountered.

First, an average push will probably be required to maintain an air speed near that already trimmed for. The force of this push will depend on the stability characteristics of the aircraft and on its c.g. position and average I.A.S. at the time. It may be considerable and it will be confusing. It will tend to contradict the unconscious reliance on feel which a pilot cannot help using to confirm his responses to instrument indications.

Secondly, a lateral gust which deflects the rudder will produce an instinctive response to centralise it. I know from experience that this response is prompt and positive. If the gust is sustained for longer than, say, one second, then a second conscious response requires that rudder be applied in the opposite direction to the effort already being exerted. This is most confusing.

To illustrate: suppose an aircraft to encounter a sustained gust from the left. The rudder will blow over to the right, and left rudder effort will be applied instinctively to centralise it. But the gust from the left will be causing the aircraft to yaw to the left, and this will be indicated on instruments and call for a correcting application of right rudder at the very time that the left foot is already pushing.

With both elevator and rudder controls playing tricks like these, the pilot may be excused for being a little unhappy. I have mentioned these rather curious effects to show that there is a definite hazard of confusion implicit in flight through turbulence; a hazard which I see no way of countering except by a thorough understanding of the behaviour likely to be experienced.

Ailerons are rather different. They play no tricks, and also, by what seems to be common consent among pilots, they are handled in a rather leisurely, even slovenly, manner. If a gust knocks a wing up then it does no harm and the next gust will probably knock it down again—if not, there is plenty of time to bring it down later.

There is sound reason behind this attitude. Ailerons coarsely used apply large twisting stresses to the wing, and at a time when the wing has enough to do in accepting the buffets of turbulence there seems little point in torturing it by twisting it as well if this does not seem essential.

Again, it is quite evidently accepted as proper practice in theory that use of ailerons, at even moderate speeds, will be restricted, as witness the considerable concessions in deflection and loading which are expressly permitted in the I.C.A.O. Standards for Recommended Procedures. (*See* I.C.A.O. Annex.8, 3.3.3.1.1.)

It is at precisely this point that the magnitude of the horizontal gusts encountered creates an objection to any angle of bank, because a horizontal gust will impose, on a banked aircraft, a relative vertical load. The vertical load imposed will be proportional to the velocity of the

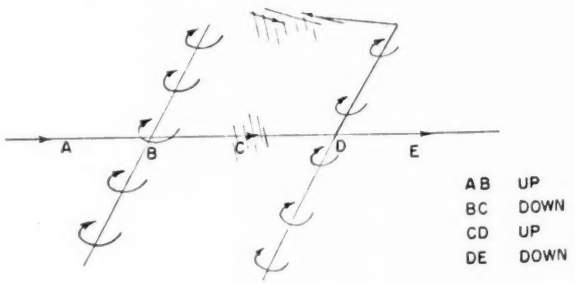

FIGURE 6.

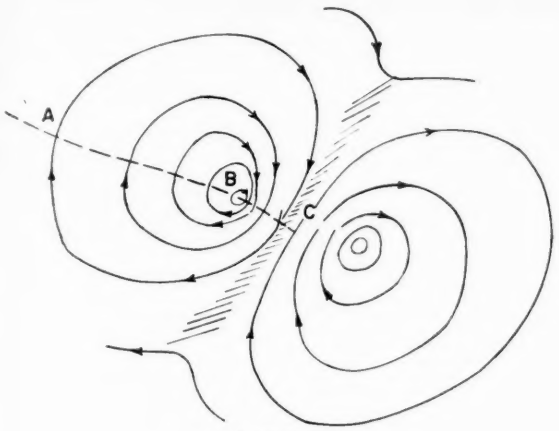

FIGURE 7.

gust times the sine of angle of bank. As an example, a 150 ft./sec. gust acting on an aircraft banked towards, or away, from it at 20 degrees will be equivalent to a 50 ft./sec. vertical gust; but the effects cease to be comparable from the point of view of the structure as the angle of bank increases. You will have noted already the small strength required to meet the down-gust requirements. This is because gravity alleviates the effect of a downward acceleration. But if the extreme case is considered wherein an aircraft banked vertically is struck by a horizontal gust, then clearly there is no such alleviation, and yet the strength available to withstand the imposed stress remains small. This leads to the suggestion that a horizontal gust which causes a down-load on a banked aircraft may very easily create a critical condition.

In view of this, it would appear advisable to regard lateral level as of equal importance to longitudinal attitude, and to strive to maintain it by as coarse use of ailerons as may be necessary, regardless of the twisting strains thereby imposed. This may call for more severe stressing requirements.

A hypothetical sequence, which could easily occur, may be of interest. If an aircraft with a large effective dihedral is imagined to encounter a pair of vortices in the manner sketched in Fig. 7, the following sequence may take place: From *A* to *B*, an increasing cross-wind from the right would be compensated for easily with aileron. At *B* a violent cross-wind from the left would strike the aircraft, and tend to roll it to the right through the region *BC*, for perhaps two seconds. The rate of roll would depend primarily on the design factors of the aircraft, and may well be greater than that which could be produced in the opposite sense by ailerons. Also, time delay in applying corrective control may permit the aircraft to roll through a considerable angle before correction is even attempted. At *C* a second violent reversal of horizontal velocity would occur which would cause a down-load on the banked wings.

As the aircraft could well have been rolled through a large angle in the region *BC*, and as the horizontal shear at *C* appears to be capable of reaching very large values, it is clear that the load imposed may be far in excess of the designed strength.

CONCLUSIONS

To sum up: I have endeavoured to offer reasonable theoretical arguments in explanation of observed phenomena and to probe some of the more obvious consequences which flow therefrom. From the aspect of operational safety, it seems that two questions may be asked:—

First, are present stressing and control standards sufficiently rigorous, particularly as they concern:—

1. Lateral gust case?
2. Response in roll to lateral gust?
3. Aileron control, and wing deformation as a result of use of ailerons?

Secondly, is present piloting technique which regards lateral level as of secondary importance, safe? Or would more rigorous control, with its acceptance of higher wing twisting stresses concurrent with vertical acceleration loads, be safer?

Note on the Bending of a Finite Rectangular Plate under Continuous Non-Normal Loading

By

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THE problems discussed here are those of a uniformly inclined and skew loadings on a finite rectangular plate such as would be obtained by the action of wind or rain on a large door of an aircraft hangar or other such building. Such a force may be considered constant in magnitude over the area of the plate but non-normal in its direction with the plate.

THE DIFFERENTIAL EQUATION:

Consider a small rectangular element of area $dxdy$. Let M_y and M_x be the bending moments per unit length and M_{yx} and M_{xy} the twisting moments per unit length acting along the edges of the element parallel to Ox and Oy respectively (*see* Fig. 2). Let Z_2 and Z_1 be the vertical shearing forces per unit length along the edges of the element parallel to Ox and Oy respectively and let X and Y be the forces per unit length acting on the element in the directions Ox and Oy.

If then F is the load on the plate per unit area with components R, P, Q in the x-, y- and z-directions, and S and S' are the horizontal reactions per unit length

Received 27th January 1955.

at the hinged edges of the plate, taking moments about Ox and Oy for the element, the following equations are obtained

$$\left.\begin{aligned}\frac{\partial M_y}{\partial y}-\frac{\partial M_{xy}}{\partial x}+Y\frac{\partial w}{\partial y}-Z_2=0\\ \frac{\partial M_x}{\partial x}+\frac{\partial M_{yx}}{\partial y}+X\frac{\partial w}{\partial x}-Z_1=0\end{aligned}\right\} \quad . \quad (1)$$

Also equating the total vertical shearing forces on the edges of the element to the downward load on it:

$$\frac{\partial Z_1}{\partial x}+\frac{\partial Z_2}{\partial y}+Q=0 \quad . \quad . \quad . \quad (2)$$

whence eliminating Z_1 and Z_2 from equations (1) and (2), and noting that $M_{xy}=-M_{yx}$

$$\frac{\partial^2 M_x}{\partial x^2}-2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_y}{\partial y^2}=-\left\{Q+\frac{\partial}{\partial x}\left(X\frac{\partial w}{\partial x}\right)+\frac{\partial}{\partial y}\left(Y\frac{\partial w}{\partial y}\right)\right\}$$

Also we see from Fig. 1 that $X=-Rx+S$ and $Y=-Py$ and the bending and twisting moments are given by:

$$M_x=-D\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right),\quad M_y=-D\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right),$$

$$M_{xy}=D(1-\nu)\frac{\partial^2 w}{\partial x\partial y}$$

where D is the plate modulus and w the downward deflection of the plate; whence the differential equation of the deflection of the plate under the general continuous loading is

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{1}{D}\left[Q+(S-Rx)\frac{\partial^2 w}{\partial x^2}-R\frac{\partial w}{\partial x}-Py\frac{\partial^2 w}{\partial y^2}-P\frac{\partial w}{\partial y}\right] \quad . \quad . \quad (3)$$

SOLUTION OF THE DIFFERENTIAL EQUATION FOR $P=0$:

First consider the case $P=0$, so that the loading becomes inclined as distinct from skew and equation (3) reduces to:

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{1}{D}\left[Q+(S-Rx)\frac{\partial^2 w}{\partial x^2}-R\frac{\partial w}{\partial x}\right] \quad . \quad (4)$$

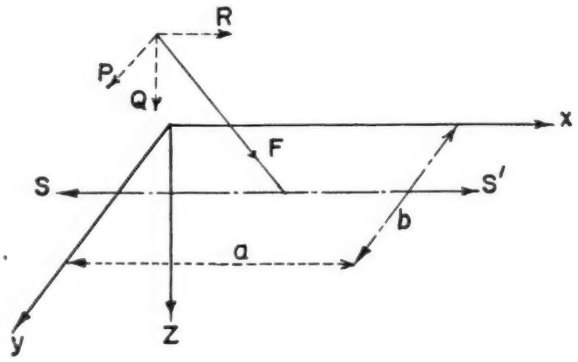

FIGURE 1.

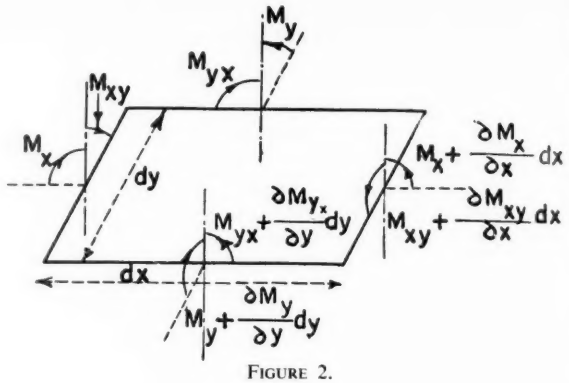

FIGURE 2.

Assuming the edges $x=0$ and $x=a$ to be freely hinged or clamped, so that they remain horizontal, a solution is required which is zero for $x=0$ and $x=a$, and so try $w=\sum_1^\infty Y_m\sin(m\pi x/a)$ where Y_m is a function of y only.

Substituting in (4)

$$\sum_1^\infty\left[Y_m''''-2\frac{m^2\pi^2}{a^2}Y_m''+\frac{m^4\pi^4}{a^4}Y_m\right]\sin\frac{m\pi x}{a}=\frac{Q}{D}+\left(\frac{S}{D}-\frac{Rx}{D}\right)\sum_1^\infty\left(\frac{m^2\pi^2}{a^2}Y_m\sin\frac{m\pi x}{a}\right)-\frac{R}{D}\sum_1^\infty\left(\frac{m\pi}{a}Y_m\cos\frac{m\pi x}{a}\right)$$

and using the Fourier expansions:

$$1=\frac{4}{\pi}\sum_1^\infty\frac{1}{2r-1}\sin\frac{(2r-1)\pi x}{a};$$

$$x=\frac{a}{2}-\frac{4a}{\pi^2}\sum_1^\infty\frac{1}{(2r-1)^2}\cos\frac{(2r-1)\pi x}{a}$$

and equating the coefficients of $\sin(m\pi x/a)$:

$$Y_m''''-\frac{2m^2\pi^2}{a^2}Y_m''+\frac{m^4\pi^4}{a^4}Y_m+\left(\frac{S}{D}-\frac{Ra}{2D}\right)\frac{m^2\pi^2}{a^2}Y_m+\frac{2R}{aD}\left[\sum_1^\infty\frac{m(m+2r-1)}{(2r-1)^2}Y_{m+2r-1}+\sum_1^\infty\frac{m(m-2r+1)}{(2r-1)^2}Y_{m-2r+1}\right]=\frac{4Q\delta_m}{\pi Dm} \quad . \quad (5)$$

where $\delta_m=0$ if m is even and $\delta_m=1$ if m is odd.

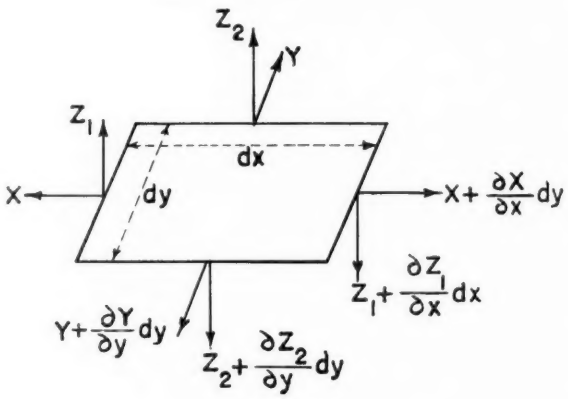

FIGURE 3.

Writing $Y_{-m} = -Y_m$, when $m=1$ (5) becomes:

$$Y_1'''' - \frac{2\pi^2}{a^2} Y_1'' + \left[\frac{\pi^4}{a^4} + \left(\frac{S}{D} - \frac{Ra}{2D}\right)\frac{\pi^2}{a^2}\right] Y_1 + \\ + \frac{2R}{aD} \sum_{(n\ \text{odd})} \frac{1\,(1+n)}{n^2} Y_{1+n} = \frac{4Q}{\pi D} \quad . \quad . \quad (6)$$

For m odd; subtracting m times equation (5) from equation (6):

$$Y_1'''' - \frac{2\pi^2}{a^2} Y_1'' + \left[\frac{\pi^4}{a^4} + \left(\frac{S}{D} - \frac{Ra}{2D}\right)\frac{\pi^2}{a^2}\right] Y_1 - \\ - m\left[Y_m'''' - \frac{2m^2\pi^2}{a^2} Y_m'' + \left\{\frac{m^4\pi^4}{a^4} + \right.\right. \\ \left.\left. + \left(\frac{S}{D} - \frac{Ra}{2D}\right)\frac{m^2\pi^2}{a^2}\right\} Y_m\right] + \frac{2R}{aD} \sum_{(n\ \text{odd})} \left[\frac{1\,(1+n)}{n^2} Y_{1+n} - \right. \\ \left. - \frac{m^2\,(m+n)}{n^2} Y_{m+n}\right] = 0 \quad . \quad . \quad (7)$$

This suggests taking $Y_m = a_m \sin(\pi y/b)$ for the case in which the edges $y=0$ and $y=b$ are also horizontal, so that writing $a_{-m} = -a_m$, and substituting in (7):

$$a_1\left[\frac{\pi^4}{b^4} + \frac{2\pi^4}{a^2b^2} + \frac{\pi^4}{b^4} + \left(\frac{S}{D} - \frac{Ra}{2D}\right)\frac{\pi^2}{a^2}\right] - \\ - ma_m\left[\frac{\pi^4}{b^4} + \frac{2m^2\pi^4}{a^2b^2} + \frac{m^4\pi^4}{a^4} + \left(\frac{S}{D} - \frac{Ra}{2D}\right)\frac{m^2\pi^2}{a^2}\right] + \\ + \frac{2R}{aD} \sum_{(n\ \text{odd})} \left[\frac{1+n}{n^2} a_{1+n} - \frac{m^2\,(m+n)}{n^2} a_{m+n}\right] = 0 \quad (8)$$

For m even; from equation (5):

$$a_m\left[\frac{\pi^4}{b^4} + \frac{2m^2\pi^2}{a^2b^2} + \frac{m^4\pi^4}{a^4} + \left(\frac{S}{D} - \frac{Ra}{2D}\right)\frac{m^2\pi^2}{a^2}\right] + \\ + \frac{2R}{aD} \sum_{(n\ \text{odd})} \frac{m\,(m+n)}{n^2} a_{m+n} = 0 \quad . \quad (9)$$

The various coefficients can then be found in terms of a_1 by solving equations (8) and (9) iteratively and substituting in equation (6) gives an equation for a_1 which can be solved graphically.

SOLUTION OF THE DIFFERENTIAL EQUATION FOR $R=0$:

This equation admits of a similar solution to that for $P=0$.

Putting $w = \sum_1^\infty a_m \sin(\pi x/a)\sin(m\pi y/b)$ the equations for the coefficients a_m become:

m odd

$$a_1\left[\frac{\pi^4}{a^4} + \left(\frac{2\pi^2}{b^2} + \frac{S}{D}\right)\frac{\pi^2}{a^2} + \frac{\pi^4}{b^4} - \frac{P\pi^2}{2bD}\right] - \\ - ma_m\left[\frac{\pi^4}{a^4} + \left(2\frac{m^2\pi^2}{b^2} + \frac{S}{D}\right)\frac{\pi^2}{a^2} + \frac{m^4\pi^4}{b^4} - \frac{P\pi^2m^2}{2bD}\right] + \\ + \frac{2P}{bD}\left[\sum_{(n\ \text{odd})} \frac{1+n}{n^2} a_{1+n} - \sum_{(n\ \text{odd})} \frac{m^2\,(m+n)}{n^2} a_{m+n}\right] = 0 \quad (10)$$

m even

$$a_m\left[\frac{\pi^4}{a^4} + \left(\frac{2m^2\pi^2}{b^2} + \frac{S}{D}\right)\frac{\pi^2}{a^2} + \frac{m^4\pi^4}{b^4} - \frac{Pm^2\pi^2}{2bD}\right] + \\ + \frac{2P}{bD} \sum_{(n\ \text{odd})} \frac{m\,(m+n)}{n^2} a_{m+n} = 0 \quad . \quad (11)$$

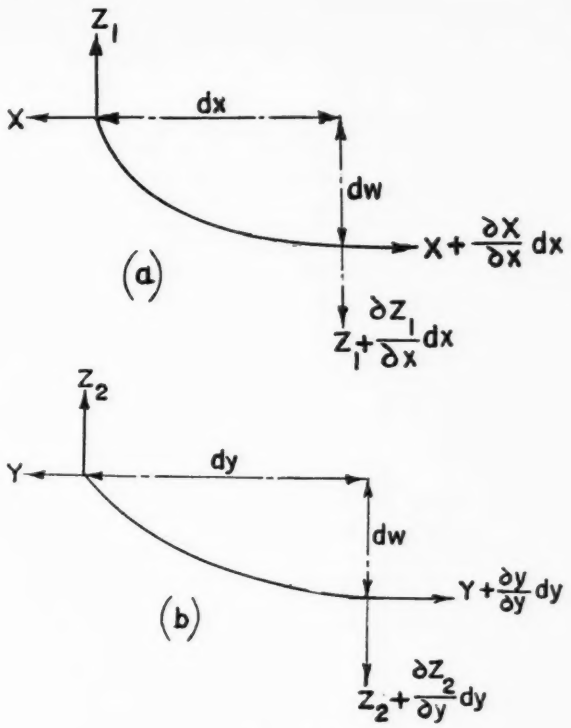

FIGURE 4.

As before the various a's are found in terms of a_1 and a_1 is solved for graphically.

THE GENERAL EQUATION:

The general equation (3) may be solved (for the case in which all edges are kept horizontal), by assuming a solution of the form:

$$w = \sum_1^\infty \sum_1^\infty a_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

which satisfies the various boundary conditions. Substituting as before and expanding q/D in a double Fourier Series as:

$$\frac{q}{D} = \frac{16q}{\pi^2 D} \sum_{(m,\,n\ \text{odd})} \sum \frac{1}{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

the coefficients are found to be given by the equations

$$a_{mn}\left[\frac{m^4\pi^4}{a^4} + \frac{2m^2n^2\pi^4}{a^2b^2} + \frac{n^4\pi^4}{b^4} + \left(\frac{S}{D} - \frac{Ra}{2D}\right)\frac{m^2\pi^2}{a^2} - \right. \\ \left. - \frac{Pn^2\pi^2}{2bD}\right] + \frac{2R}{aD} \sum_{(r\ \text{odd})} \left[a_{m-r,\,n} \frac{m\,(m-r)}{r^2} + a_{m+r,\,n} \frac{m\,(m+r)}{r^2}\right] + \\ + \frac{2P}{bD} \sum_{(r\ \text{odd})} \left[a_{m,\,n-r} \frac{n\,(n-r)}{r^2} + a_{m,\,n+r} \frac{n\,(n+r)}{r^2}\right] = \\ = \frac{16Q}{\pi^2 D} \frac{1}{mn} \delta_m \delta_n$$

where δ_m, $\delta_n = 0$ for m, n even, and δ_m, $\delta_n = 1$ for m, n odd; $a_{-m,\,n} = -a_{m,\,n}$, and $a_{m,\,-n} = -a_{m,\,n}$.

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Reviews

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THE MOLECULAR THEORY OF GASES AND LIQUIDS. *J. O. Hirschfelder, C. F. Curtiss and R. B. Bird. John Wiley, New York. Chapman & Hall, London, 1954. 1,219 pp. Diagrams. 160s.*

The aerodynamicist is so used to regarding fluids as continua, that this book may at first appear to him as subversive. This impression is likely to give way to surprise at, and ultimately respect for, the extensive and highly-developed theory, built up quite independently of aerodynamics, which starts from the fact that fluids are not continua but consist of discrete particles, a theory which is lucidly and exhaustively presented in this authoritative volume. The molecular theory of liquids and gases has two main branches: the study of equilibrium properties such as the pressure-volume-temperature relations of substances; and the study of such non-equilibrium processes as viscous friction, thermal conduction, and molecular diffusion. These branches have a common root and explanation in the study of the forces which neighbouring molecules exert on each other. Since very small quantities of energy are involved, classical mechanics is inadequate for any but approximate treatments, and quantum-mechanical methods must be used. The hesitant reader may be comforted however to learn that relativistic concepts need not be invoked.

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Aeronautical engineers are unlikely to do more than dip into this book for many years to come. Whenever excursions into the molecular fields are inescapable, however, this is the place to start from.—D. B. SPALDING (Imperial College).

FUNDAMENTALS OF TRANSISTORS. *Leonard M. Krugman. John F. Rider, New York. Chapman & Hall, London, 1954. 140 pp. Illustrations. 21s.*

This is a practical book and deals with transistors for the man who, for work or play, not only designs but builds his own circuits. The transistor replaces the thermionic valve in many applications having the advantages of weight, size, power requirement, efficiency and life. It can be subjected to accelerations and vibrations not permissible in a valve. For these reasons it is inevitable that it will be extensively used in electronic equipment fitted in aircraft.

There are two aspects to transistor knowledge, the first is the physical mechanism of the internal operation. This is extremely complex and much of the phenomena is not understood. The second uses the transistor properties and characteristics and shows how electronic circuits may be built around them. In this book advanced physical and mathematical ideas necessary to understand the internal operation are completely eliminated and replaced by a short over-simplified description. The emphasis of the book is on describing the now standard forms of transistors and the types of circuitry which have been evolved. Whole chapters are devoted to amplifiers and oscillators, *i.e.* the units which go to form the major part of most electronic apparatus.

The book is of most use to engineering students requiring initial knowledge of the principles and practice of this new device. It will also be of value to a much wider circle of readers who are interested to learn of the extent of the practical circuit applications.

The addition of even a small bibliography of more advanced and original work in this field would have been helpful.—JOHN C. WEST.

TECHNISCHE HYDRO-UND AEROMECHANIK. *Walther Kaufmann. Springer, Berlin, 1954. 352 pp. Diagrams. (In German.) D.M. 36 (60s. approx.).*

Professor Kaufmann's intention in this book is to expound the fundamental concepts of fluid motion, together with their most important technical applications, in a book that is of moderate length and is suitable for engineering students at *Technische Hochschulen*. He stresses the importance of boundary layer theory in closing the gap between the classical theory of inviscid flow and hydraulics. One need only reflect on the amount of material and on the mathematical apparatus to be found, say, in Lamb's *Hydrodynamics* and in the four volumes of *Modern Developments* to realise how difficult a task this is, but Professor Kaufmann tackles it with a marked degree of success.

Two short introductory chapters on the nature of fluids and on hydrostatics are followed by the main material, which is divided into three principal parts. In the first part (93 pp.) the equations of one-dimensional flow are derived and applied. The effect of viscosity on the flow in pipes and channels is then described; the parabolic distributions of laminar flow are followed by Prandtl's and Kármán's momentum mixture length theory for turbulent flow. In the second part (192 pp.) the equations of inviscid three-dimensional flow are derived and the subjects of potential flow, conformal transformation, circulation, surface waves, and vorticity are introduced. There follow substantial sections on the Navier-Stokes equations, on boundary layer theory and on the flow past wings, cascades and airscrews. The third part (35 pp.) is a brief introduction to compressible flow: the basic differential equations are followed by sections on steady one-dimensional flow, on linearised two-dimensional flow, and on the method of characteristics for two-dimensional isentropic flow.

The many examples of applications of the fundamental equations to practical problems are of a high standard. The exposition is in general lucid and precise, and is greatly enhanced by more than 250 detailed and excellent diagrams.

It is easy to point out gaps in the material—no exact solutions of the Navier-Stokes equations are given, although considerable space is given to solutions which neglect the inertia terms; no mention is made of surface distributions of sources and doublets; and free-streamline flows are ignored—but such objections may be unjust in view of the introductory nature of this book. A desire for simplicity may also account for the fact that vector analysis is introduced only spasmodically, and is by no means exploited fully; but where it does appear it is obscured, in my opinion, by the use of " div," " grad " and " rot " instead of the operator V. The arrangement of the material may be open to mild criticism: it would seem desirable that the student became familiar with inviscid flow in more than one dimension before he is faced with such concepts as mixture length theory—but opinions on this point may differ.

On the whole the book provides engineers with an extremely sound introduction to fluid mechanics.—L. E. FRAENKEL (Imperial College).

TABLE OF SINE AND COSINE INTEGRALS FOR ARGUMENTS FROM 10 TO 100. *National Bureau of Standards Applied Mathematics Series 32. Government Printing Office, Washington 25 D.C., 1954. 187 pp. $2.25.*

The values of the integrals

$$\mathrm{Si}\,(x)=\int_0^x \frac{\sin u}{u}\,du \quad \text{and} \quad \mathrm{Ci}\,(x)=\int_\infty^x \frac{\cos u}{u}\,du$$

are tabulated to 10 decimal places for the range of x between 10 and 100 at intervals of 0·01. The tables thus cover a wider range with a smaller interval of the argument than has previously been published. In addition, four pages at the end of the book contain tables of multiples of $\pi/2$, and of the functions $\frac{1}{2}p(1-p)$, $p(1-p^2)/6$ and $q(1-q^2)/6$ where $p+q=1$, to facilitate interpolation in the main tables.

Additions to the Library

A.I.A. (U.S.). NATIONAL AIRCRAFT STANDARDS.

Battelle Memorial Institute. PREVENTION OF THE FAILURE OF METALS UNDER REPEATED STRESS. 1949.

B.S.I. BAROMETER CONVENTIONS AND TABLES. B.S.2520. B.S.I. 1954.

Dudding, B. P. and W. J. Jennett. CONTROL CHART TECHNIQUE, WHEN MANUFACTURING TO A SPECIFICATION. B.S.2564. B.S.I. 1955.

Ellis, F. H. CANADA'S FLYING HERITAGE. University of Toronto Press. 1954.

Fedden, Sir Roy. A CENTURY OF PROGRESS IN AERONAUTICS. (Reprint: *Trans. Soc. Eng.*) Reprint. 1954.

Geary, P. J. FLEXURE DEVICES: PIVOTS, MOVEMENTS, SUSPENSIONS. A BIBLIOGRAPHICAL SURVEY. B.S.I.R.A. 1954.

Green, W. and R. Cross. THE JET AIRCRAFT OF THE WORLD. Macdonald. 1955.

Judge, A. W. THE TESTING OF HIGH SPEED INTERNAL COMBUSTION ENGINES. 4th edition revised. Chapman and Hall. 1955.

Krugman, L. M. FUNDAMENTALS OF TRANSISTORS. Chapman and Hall. 1954.

Lawrence, T. E. THE MINT. Cape. 1955.

M.O.T. & C.A. REPORT OF THE COURT OF INQUIRY INTO THE ACCIDENTS TO COMET G-ALYP AND COMET G-ALYY. H.M.S.O. 1955.

National Aircraft Standards Committee. REPORT OF THE FIRST NATIONAL SYMPOSIUM FOR TITANIUM STANDARD PARTS. Aircraft Industries Assoc. 1954.

Navy Training Publications Center. AIRCRAFT PROPELLERS. U.S.G.P.O. 1954.

N.P.L. MEASUREMENT OF PRESSURE WITH THE MERCURY BAROMETER. (NOTES ON APPLIED SCIENCE NO. 9.) H.M.S.O. 1955.

Rexford-Welch, S. C. (*Editor*). THE ROYAL AIR FORCE MEDICAL SERVICES, VOL. I—ADMINISTRATION. H.M.S.O. 1954.

Sell, G. (*Editor*). THE ROLE OF PETROLEUM IN MODERN TRANSPORT. Institute of Petroleum. 1955.

Sperry Gyroscope Co. A FLIGHT INVESTIGATION OF THE PERFORMANCE OF LOW CEILING/VISIBILITY METEOROLOGICAL EQUIPMENT. 1954.

Woodson, W. E. HUMAN ENGINEERING GUIDE FOR EQUIPMENT DESIGNERS. University of California Press. 1954.

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The aerodynamicist is so used to regarding fluids as continua, that this book may at first appear to him as subversive. This impression is likely to give way to surprise at, and ultimately respect for, the extensive and highly-developed theory, built up quite independently of aerodynamics, which starts from the fact that fluids are not continua but consist of discrete particles, a theory which is lucidly and exhaustively presented in this authoritative volume. The molecular theory of liquids and gases has two main branches: the study of equilibrium properties such as the pressure-volume-temperature relations of substances; and the study of such non-equilibrium processes as viscous friction, thermal conduction, and molecular diffusion. These branches have a common root and explanation in the study of the forces which neighbouring molecules exert on each other. Since very small quantities of energy are involved, classical mechanics is inadequate for any but approximate treatments, and quantum-mechanical methods must be used. The hesitant reader may be comforted however to learn that relativistic concepts need not be invoked.

A long chapter is devoted to hydrodynamical applications of molecular theory, comprising the various conservation laws, irreversible thermodynamics, energy transfer by radiation, sound propagation, unsteady one-dimensional compressible flow, the propagation of steady flames, shock waves, detonations, and gas flow in rockets. Although much of this is available in greater detail elsewhere—and moreover in a form more readily assimilable by the engineer—the present treatment has the great virtue of linking these subjects with each other and with the molecular theory. The authors have attempted to show the essential unity of the apparently separate fields of study, and so, though the first impression is quite otherwise, may be found to have achieved a prodigious task of simplification.

Aeronautical engineers are unlikely to do more than dip into this book for many years to come. Whenever excursions into the molecular fields are inescapable, however, this is the place to start from.—D. B. SPALDING (Imperial College).

FUNDAMENTALS OF TRANSISTORS. *Leonard M. Krugman. John F. Rider, New York. Chapman & Hall, London, 1954. 140 pp. Illustrations. 21s.*

This is a practical book and deals with transistors for the man who, for work or play, not only designs but builds his own circuits. The transistor replaces the thermionic valve in many applications having the advantages of weight, size, power requirement, efficiency and life. It can be subjected to accelerations and vibrations not permissible in a valve. For these reasons it is inevitable that it will be extensively used in electronic equipment fitted in aircraft.

There are two aspects to transistor knowledge, the first is the physical mechanism of the internal operation. This is extremely complex and much of the phenomena is not understood. The second uses the transistor properties and characteristics and shows how electronic circuits may be built around them. In this book advanced physical and mathematical ideas necessary to understand the internal operation are completely eliminated and replaced by a short over-simplified description. The emphasis of the book is on describing the now standard forms of transistors and the types of circuitry which have been evolved. Whole chapters are devoted to amplifiers and oscillators, *i.e.* the units which go to form the major part of most electronic apparatus.

The book is of most use to engineering students requiring initial knowledge of the principles and practice of this new device. It will also be of value to a much wider circle of readers who are interested to learn of the extent of the practical circuit applications.

The addition of even a small bibliography of more advanced and original work in this field would have been helpful.—JOHN C. WEST.

TECHNISCHE HYDRO-UND AEROMECHANIK. *Walther Kaufmann. Springer, Berlin, 1954. 352 pp. Diagrams. (In German.) D.M. 36 (60s. approx.).*

Professor Kaufmann's intention in this book is to expound the fundamental concepts of fluid motion, together with their most important technical applications, in a book that is of moderate length and is suitable for engineering students at *Technische Hochschulen*. He stresses the importance of boundary layer theory in closing the gap between the classical theory of inviscid flow and hydraulics. One need only reflect on the amount of material and on the mathematical apparatus to be found, say, in Lamb's *Hydrodynamics* and in the four volumes of *Modern Developments* to realise how difficult a task this is, but Professor Kaufmann tackles it with a marked degree of success.

Two short introductory chapters on the nature of fluids and on hydrostatics are followed by the main material, which is divided into three principal parts. In the first part (93 pp.) the equations of one-dimensional flow are derived and applied. The effect of viscosity on the flow in pipes and channels is then described; the parabolic distributions of laminar flow are followed by Prandtl's and Kármán's momentum mixture length theory for turbulent flow. In the second part (192 pp.) the equations of inviscid three-dimensional flow are derived and the subjects of potential flow, conformal transformation, circulation, surface waves, and vorticity are introduced. There follow substantial sections on the Navier-Stokes equations, on boundary layer theory and on the flow past wings, cascades and airscrews. The third part (35 pp.) is a brief introduction to compressible flow: the basic differential equations are followed by sections on steady one-dimensional flow, on linearised two-dimensional flow, and on the method of characteristics for two-dimensional isentropic flow.

The many examples of applications of the fundamental equations to practical problems are of a high standard. The exposition is in general lucid and precise, and is greatly enhanced by more than 250 detailed and excellent diagrams.

It is easy to point out gaps in the material—no exact solutions of the Navier-Stokes equations are given, although considerable space is given to solutions which neglect the inertia terms; no mention is made of surface distributions of sources and doublets; and free-streamline flows are ignored—but such objections may be unjust in view of the introductory nature of this book. A desire for simplicity may also account for the fact that vector analysis is introduced only spasmodically, and is by no means exploited fully; but where it does appear it is obscured, in my opinion, by the use of "div," "grad" and "rot" instead of the operator V. The arrangement of the material may be open to mild criticism: it would seem desirable that the student became familiar with inviscid flow in more than one dimension before he is faced with such concepts as mixture length theory—but opinions on this point may differ.

On the whole the book provides engineers with an extremely sound introduction to fluid mechanics.—L. E. FRAENKEL (Imperial College).

TABLE OF SINE AND COSINE INTEGRALS FOR ARGUMENTS FROM 10 TO 100. *National Bureau of Standards Applied Mathematics Series 32. Government Printing Office, Washington 25 D.C., 1954. 187 pp. $2.25.*

The values of the integrals

$$\mathrm{Si}\,(x)=\int_0^x \frac{\sin u}{u}\,du \quad \text{and} \quad \mathrm{Ci}\,(x)=\int_\infty^x \frac{\cos u}{u}\,du$$

are tabulated to 10 decimal places for the range of x between 10 and 100 at intervals of 0·01. The tables thus cover a wider range with a smaller interval of the argument than has previously been published. In addition, four pages at the end of the book contain tables of multiples of $\pi/2$, and of the functions $\frac{1}{2}p(1-p)$, $p(1-p^2)/6$ and $q(1-q^2)/6$ where $p+q=1$, to facilitate interpolation in the main tables.

Additions to the Library

A.I.A. (U.S.). NATIONAL AIRCRAFT STANDARDS.

Battelle Memorial Institute. PREVENTION OF THE FAILURE OF METALS UNDER REPEATED STRESS. 1949.

B.S.I. BAROMETER CONVENTIONS AND TABLES. B.S.2520. B.S.I. 1954.

Dudding, B. P. and W. J. Jennett. CONTROL CHART TECHNIQUE, WHEN MANUFACTURING TO A SPECIFICATION. B.S.2564. B.S.I. 1955.

Ellis, F. H. CANADA'S FLYING HERITAGE. University of Toronto Press. 1954.

Fedden, Sir Roy. A CENTURY OF PROGRESS IN AERONAUTICS. (Reprint: *Trans. Soc. Eng.*) Reprint. 1954.

Geary, P. J. FLEXURE DEVICES: PIVOTS, MOVEMENTS, SUSPENSIONS. A BIBLIOGRAPHICAL SURVEY. B.S.I.R.A. 1954.

Green, W. and R. Cross. THE JET AIRCRAFT OF THE WORLD. Macdonald. 1955.

Judge, A. W. THE TESTING OF HIGH SPEED INTERNAL COMBUSTION ENGINES. 4th edition revised. Chapman and Hall. 1955.

Krugman, L. M. FUNDAMENTALS OF TRANSISTORS. Chapman and Hall. 1954.

Lawrence, T. E. THE MINT. Cape. 1955.

M.O.T. & C.A. REPORT OF THE COURT OF INQUIRY INTO THE ACCIDENTS TO COMET G-ALYP AND COMET G-ALYY. H.M.S.O. 1955.

National Aircraft Standards Committee. REPORT OF THE FIRST NATIONAL SYMPOSIUM FOR TITANIUM STANDARD PARTS. Aircraft Industries Assoc. 1954.

Navy Training Publications Center. AIRCRAFT PROPELLERS. U.S.G.P.O. 1954.

N.P.L. MEASUREMENT OF PRESSURE WITH THE MERCURY BAROMETER. (NOTES ON APPLIED SCIENCE NO. 9.) H.M.S.O. 1955.

Rexford-Welch, S. C. (*Editor*). THE ROYAL AIR FORCE MEDICAL SERVICES, VOL. I—ADMINISTRATION. H.M.S.O. 1954.

Sell, G. (*Editor*). THE ROLE OF PETROLEUM IN MODERN TRANSPORT. Institute of Petroleum. 1955.

Sperry Gyroscope Co. A FLIGHT INVESTIGATION OF THE PERFORMANCE OF LOW CEILING/VISIBILITY METEOROLOGICAL EQUIPMENT. 1954.

Woodson, W. E. HUMAN ENGINEERING GUIDE FOR EQUIPMENT DESIGNERS. University of California Press. 1954.

Reports

AERODYNAMICS

Boundary Layer

Shearing-stress measurements by use of a heated element. H. W. Liepmann and G. T. Skinner. N.A.C.A. Technical Note 3268 (*November* 1954).

The present report discusses the use of an instrument to determine the local skin-friction coefficient by measurement of heat transfer from small elements embedded in the surface. The range of application of such instruments is discussed and experimental data are presented to show that a simple instrument consisting of an ordinary hot-wire cemented into a groove in the surface can be used to obtain laminar and turbulent skin-friction coefficients with a single calibration.—(1.1.0).

Boundary-layer transition at Mach 3·12 *with and without single roughness elements. P. F. Brinich. N.A.C.A. Technical Note* 3267 (*December* 1954).

Transition on a hollow cylinder aligned with the air stream was studied in a Mach 3·12 wind tunnel at Reynolds numbers from 1×10^5 to 7×10^5 per inch. Transition was observed from surface-temperature distributions and Schlieren photographs with and without single roughness elements.—(1.1.2.4).

An analytical estimation of the effect of transpiration cooling on the heat-transfer and skin-friction characteristics of a compressible turbulent boundary layer. M. W. Rubesin. N.A.C.A. Technical Note 3341 (*December* 1954).

This analysis based on mixing-length theory indicates that surface blowing associated with transpiration cooling systems produces large reductions in both the heat-transfer and skin-friction coefficients for a turbulent boundary layer on a flat plate. The numerical results are restricted to the case of air blowing into air.—(1.1.3 × 1.9.1).

Control Surfaces

See also Propellers

The delay of turbulent separation on a flap by the use of air jets. G. A. Collins. A.R.L. Aerodynamics Note 129 (*November* 1953).

The effectiveness of air jets for turbulent boundary layer control has been investigated on the upper surface of an aerofoil with a plain flap at a Reynolds number of $0{\cdot}35\times10^6$. Changes in lift and drag brought about by the improved flow, due to the delay of turbulent separation, have been measured together with the flow quantity through the jets.—(1.3.4).

Fluid Dynamics

Uber die Temperaturverteilung hinter angeströmten Zylindern. J. Ackeret.—(1.4).

Uber thermische Effekte in Resonanzrohren. H. Sprenger.—(1.9).

Schubvermehrung durch Strahlmischung. Z. Plaskowski.—(29.6).

Mitteilungen aus dem Institut fur Aerodynamik. Mitteilungen aus dem Institut fur Aerodynamik. Zurich. No. 21.—(1954).

Internal Flow

Measurement of the flow through small holes. G. F. Ward. A.R.L. Aerodynamics Note 127 (*September* 1953).

Flow coefficients and quantities for small holes in various materials have been experimentally established for conditions within the following limits—(*i*) Reynolds numbers 1,000 to 14,000, (*ii*) hole diameters 0·20 in. to 0·053 in., (*iii*) pressure differences 0·5 in. Hg to 6 in. Hg.—(1.5.1.3).

Smoke study of nozzle secondary flows in a low-speed turbine. M. G. Kofskey and H. W. Allen. N.A.C.A. Technical Note 3260 (*November* 1954).

Still and motion pictures were made of boundary layer and wake secondary-flow phenomena visualised by smoke. Two annular cascades of turbine nozzles were used, both designed for constant discharge angle but differing in blade shape and suction-surface velocity distribution.—(1.5.3.2).

Approximate effect of leading-edge thickness, incidence angle, and inlet Mach number on inlet losses for high-solidity cascades of low cambered blades. L. C. Wright. N.A.C.A. Technical Note 3327 (*December* 1954).

An approximate analysis of cascade induction loss variations with incidence angle, Mach number, and leading-edge thickness is presented with computational results in the form of curves. Results are based on the solution of the energy, continuity, and integrated momentum equations under the assumption that the blade leading-edge pressure approaches zero at appreciable incidence angles.—(1.5.4.2).

Stability and Control

See also Testing and Instruments

Theoretical calculations of the lateral stability derivatives for triangular vertical tails with subsonic leading edges travelling at supersonic speeds. P. J. Bobbitt. N.A.C.A. Technical Note 3240 (*December* 1954).

Pressure-distribution expressions and stability derivatives have been derived for zero-end-plate triangular vertical tails performing yawing, rolling, and constant-lateral-acceleration motions by a method for solving supersonic-conical flow boundary-value problems. In addition, by using the yawing and constant-lateral-acceleration results, the damping of the vertical tail oscillating laterally in yaw has been approximated to the first order in frequency.—(1.8.1.2).

A theoretical investigation of the short-period dynamic longitudinal stability of airplane configurations having elastic wings of 0° *to* 60° *sweepback. M. D. McLaughlin. N.A.C.A. Technical Note* 3251 (*December* 1954).

An analytical investigation utilising the semi-rigid approach was made to determine the effects of wing flexibility on the dynamic longitudinal stability of thin-wing aeroplane configurations. Solutions were obtained in the form of frequency and damping of the flexible wing mode and of the aeroplane short-period stability mode.—(1.8.2.1).

Thermo-aerodynamics

See also Boundary Layer and Fluid Dynamics

Theoretical and experimental investigation of aerodynamic-heating and isothermal heat-transfer parameters on a hemispherical nose with laminar boundary layer at supersonic Mach numbers. H. A. Stine and K. Wanlass. N.A.C.A. Technical Note 3344 (*December* 1954).

Measurements of the heat-transfer parameter, $Nu/\sqrt{Re}$, based on local flow conditions just outside the boundary layer and length of boundary-layer run were carried out on the heated, isothermal surface of a hemisphere-cylinder for a Mach number of 1·97 and Reynolds numbers based on body diameter from $0{\cdot}60\times10^6$ to $2{\cdot}28\times10^6$. An approximate method for calculating the distribution of heat-transfer parameter, applicable to any body of revolution, was developed.—(1.9.1).

Wings and Aerofoils

See also Propellers

The effect of surface roughness on the stalling characteristics of a thin aerofoil. D. G. Hurley and G. F. Ward. A.R.L. Aerodynamics Note 109 (*December* 1952).

Experiments were made on a model which simulated the flow conditions near the nose of an N.A.C.A. 64A006 aerofoil

NOTE.—*The figures in parenthesis at the end of each Summary are for office use only.*

at a Reynolds number of 3·85 million. It was found that the separation of the flow from the nose of the aerofoil could be postponed to larger incidence by bringing about transition in the boundary layer before the laminar separation point, by means of suitably small roughness elements located near the nose of the aerofoil. The postponement obtained in this way was about the same as that obtained using small air jets located between the front stagnation point and the leading edge.—(1.10.2.1).

Experiments on the effects of air jets and surface roughness on the boundary layer near the nose of an N.A.C.A. 64A006 aerofoil. D. G. Hurley and G. F. Ward. A.R.L. Aerodynamics Note 128 (*September* 1953).

Further experiments have been made on the nose of an N.A.C.A. 64A006 aerofoil to compare the improvements that can be obtained in its stalling characteristics using either a row of air jets or surface roughness located near the leading edge. The test Reynolds number was 3·85 million.—(1.10.2.1).

Determination of optimum air jet arrangement to delay nose stalling on the articulated nose of a thin wing. R. A. Wallis and G. F. Ward. A.R.L. Aerodynamics Note 133 (*January* 1954).

The optimum air jet configuration for suppressing laminar separation on the nose of a 10 ft. chord thin wing was determined. A single row of small holes midway between the stagnation point just prior to nose stalling and the leading edge at a pitch of $\frac{1}{4}$ in. gave best results.—(1.10.2.1).

Experiments with air jets to control the nose stall on a 3 ft. chord N.A.C.A. 64A006 aerofoil. R. A. Wallis. A.R.L. Aerodynamics Note 139 (*September* 1954).

Experiments on a N.A.C.A. 64A006 two-dimensional aerofoil have shown that air jets on the lower surface of the nose can be used to suppress the laminar separation "bubble," thereby improving the stalling characteristics of the aerofoil. In one series of tests, additional air jets on the upper surface near the leading edge effected an appreciable delay in turbulent separation from the nose.—(1.10.2.1).

Generalized indicial forces on deforming rectangular wings in supersonic flight. H. Lomax, F. B. Fuller and L. Sluder. N.A.C.A. Technical Note 3286 (*November* 1954).

The time-dependent flow is determined over a rectangular wing moving with a supersonic forward speed and undergoing small vertical distortions expressible as polynomials involving spanwise and chordwise distances. Numerical results for Mach numbers of 1·1 and 1·2 are given for polynomials of the first and fifth degree in the chordwise and spanwise directions, respectively, on a wing of aspect ratio 4.—(1.10.1.3 × 2).

An investigation of a lifting 10*-percent-thick symmetrical double-wedge airfoil at Mach numbers up to* 1. *M. D. Humphreys. N.A.C.A. Technical Note* 3306 (*November* 1954).

Subsonic and transonic pressure-distribution and drag-coefficient measurements on a two-dimensional 10-per cent.-thick symmetrical double-wedge aerofoil for a range of angles of attack are presented and compared with theory and existing experimental data.—(1.10.2.1).

Testing and Instruments

Optical considerations and limitations of the Schlieren method. G. S. Speak and D. J. Walters. R. & M. 2859 (*January* 1950, *published* 1954).

The elementary principles of the Schlieren method are first described, with reference to an ideal basic system. Various developments from this basic system and the experimental procedure in setting up the system are then considered with particular reference to the advantages and disadvantages from the optical point of view. The general optical theory of the Schlieren method is worked out, firstly in terms of the deflection of a ray which passes through a medium of varying refractive index, secondly in terms of the change in illumination caused by this deflection, which is calculated from diffraction theory. Some typical examples are worked out in the latter case.—(1.12).

Free convection of heat from fine wires. D. C. Collis and M. J. Williams. A.R.L. Aerodynamics Note 140 (*September* 1954).

The influence of length, orientation and temperature loading on the weak convective heat transfer from 0·0003 cm. diameter platinum wire has been examined.—(1.12.6.3).

A summary of the techniques of variable Mach number supersonic wind tunnel nozzle design. J. T. Kenney and L. M. Webb. AGARDograph 3 (*October* 1954).

A procedure for the aerodynamic design of flexible nozzles capable of continuous Mach number variation is developed in detail. The special structural, mechanical calibration and cost estimation problems involved in flexible nozzle construction are discussed.—(1.12.1.3).

*A scheme of automatic data reduction for wind tunnels. K. V. Diprose. Agard Memorandum AG*9/*M*5 (*September* 1953).—(1.12).

*A note on the use of strain gauges in wind tunnel balances. J. R. Anderson. Agard Memorandum AG*10/*M*6 (*September* 1953).—(1.12.4).

Use of a hot-wire anemometer in shock-tube investigations. D. S. Dosanjh. N.A.C.A. Technical Note 3163 (*December* 1954).

The response of a hot-wire to a transient step-function type of change in flow conditions is experimentally studied and a criterion for predicting the right compensation is postulated and experimentally verified. Besides using the hot-wire for actual wave phenomena and for flow studies, it has been very successfully used as a timing and/or triggering device in shock-tube work.—(1.12.6.3).

A system for measuring the dynamic lateral stability derivatives in high-speed wind tunnels. H. C. Lessing, T. B. Fryer and M. H. Mead. N.A.C.A. Technical Note 3348 (*December* 1954).

A two-degree-of-freedom system in which rolling oscillations are forced is described with details of the system, the theory, and the method of operation. Tests are described in which certain of the aerodynamic derivatives were simulated, and the results of these tests, as well as of a wind-tunnel investigation of a simple model, are presented.—(1.12 × 1.8.1).

AEROELASTICITY

See also Aerodynamics: Wings and Aerofoils

On the analysis of linear and nonlinear dynamical systems from transient-response data. M. Shinbrot. N.A.C.A. Technical Note 3288 (*December* 1954).

A general theory of the so-called "equations-of-motion" methods for linear dynamical systems is developed first. It is then shown that all these linear methods can be extended in a perfectly natural way to apply to non-linear systems. In addition, a new method, possessing certain advantages over previous methods, is derived, and demonstrated by several examples.—(2).

Initial experiments on flutter of unswept cantilever wings at Mach number 1·3. *W. J. Tuovila, J. E. Baker and A. A. Regier. N.A.C.A. Technical Note* 3312 (*November* 1954).

Comparison with the theory of flutter at supersonic speeds indicates that this theory is applicable in predicting cantilever-wing flutter. These preliminary experiments consider also various overall effects of centre-of-gravity location, elastic-axis location, mass-density parameter, section shape, and some wing-tip moments of inertia.—(2).

Comparison of flutter calculations using various aerodynamic coefficients with experimental results for some rectangular cantilever wings at Mach number 1·3. *H. C. Nelson and R. A. Rainey. N.A.C.A. Technical Note* 3301 (*November* 1954).

A general Rayleigh analysis is used as a basis for developing four methods of flutter analysis that are applied to twelve rectangular low-aspect-ratio wings with aspect ratios ranging from 3·00 to 4·55. These wings were previously tested at a Mach number of 1·3 by progressively varying certain wing parameters until flutter occurred. The four methods

of flutter analysis used are: section coefficients for harmonically pitching and translating rectangular wings in a Rayleigh type of analysis, two-dimensional coefficients in a Rayleigh type of analysis, total coefficients for harmonically pitching and translating rectangular wings in a representative-section analysis, and two-dimensional coefficients in a representative-section analysis.—(2).

AIRPORTS

A lighting pattern for runway zone identification. A. T. Tiedemann. C.A.A. Technical Development Report No. 208 (*December* 1954).

Various methods are investigated for providing a pilot with visual information for determining his longitudinal position along the runway. The runway zone-marking system which is recommended utilises additional coloured lights located in the section at each end of the runway to define a safe touch-down area on the approach end and a caution area on the far end.—(6.3).

A survey of background and aircraft noise in communities near airports. K. N. Stevens. N.A.C.A. Technical Note 3379 (*December* 1954).

An extensive survey has been made of background and aircraft noise levels in residential communities in eight cities having major airports. The measurements were made in areas up to a distance of 12 miles from the airports, and the areas were chosen to be under regularly used flight paths. Spectra were obtained from magnetic tape recordings of the noise of about 250 aircraft in flight, representing substantially all commercial types.—(6).

ELECTRONICS

See also NAVIGATION

A stabilized local-oscillator performance evaluator. A. W. Randall and O. E. McIntire. C.A.A. Technical Development Report No. 249 (*December* 1954).

An instrument designed to provide accurate quantitative data on the stability and performance characteristics of microwave oscillators is described. The application of the instrument and the interpretation of the data yielded are discussed in detail.—(11).

FLIGHT TESTING

Elements de base des mesures en vol. F. Beteille. Publications Scientifiques et Techniques du Ministère de L'Air. No. B.S.T. 117 (1954).—(13.1).

FUELS AND LUBRICANTS

*Potentiometric determination of mercaptan-sulphur in aviation turbine fuels. F. G. Kitson. N.A.E. Laboratory Report LR-*113 (*September* 1954).—(14.2).

Burning times of magnesium ribbons in various atmospheres. K. P. Coffin. N.A.C.A. Technical Note 3332 (*December* 1954).

Some details of the mechanism of the combustion of magnesium ribbons were investigated in mixture of oxygen (17 to 100 per cent.) and inert gases (A, N_2, He, A-H_2O mixtures).—(14.3).

Friction of possible solid lubricants with various crystal structures. M. B. Peterson and R. L. Johnson. N.A.C.A. Technical Note 3334 (*December* 1954).

A number of solids with CdI_2, $CdCl_2$, and MoS_2 types of layer lattice were tested in a low-speed friction apparatus for lubrication effectiveness.—(14.3).

MATERIALS

Étude des textures cristallines obtenues par laminage progressif de monocristaux d'aluminium. J. Manenc. Publications Scientifiques et Techniques du Ministère de L'Air. No. 293 (1954).—(21.2.2).

Recherches magnétochimiques sur l'anisotropie des polymeres. Jean Ploquin. Publications Scientifiques et Techniques du Ministère de L'Air. No. 295 (1954).—(21.3.3).

The effect of rate of cycling on the fatigue properties of 24S-T *aluminium alloy. J. Y. Mann. A.R.L. Report SM.*188 (*August* 1954).

Rotating cantilever fatigue tests have been made at 21, 170, 385, 600, 1,425 and 12,000 cycles per minute to determine the effect of rate of cycling on the fatigue resistance of 24S-T aluminium alloy.—(21.2.2).

Experimental investigation of notch-size effects on rotating-beam fatigue behavior of 75S-T6 *aluminum alloy. W. S. Hyler, R. A. Lewis and H. J. Grover. N.A.C.A. Technical Note* 3291 (*November* 1954).

This investigation was initiated to study the influence of size, particularly the notch size, on extruded 75S-T6 aluminium-alloy test specimens. Unnotched and notched specimens with five different minimum-section diameters were tested. For each size a semi-circular groove was tested and for the largest diameter specimen a V-notch was also tested. A method of surface preparation was selected that would produce comparable surface finishes in different-sized notched and unnotched specimens.—(21.2.2).

Influence of exposed area on stress-corrosion cracking of 24S *aluminum alloy. W. H. Colner and H. T. Francis. N.A.C.A. Technical Note* 3292 (*November* 1954).

Results are presented of a study of the "area effect" in 24S aluminium alloy. This effect is the phenomenon whereby small exposed areas show long times to stress-corrosion failure, whereas large areas show short times. The effects of stress level, degree of sensitivity of the alloy, and hydrogen peroxide concentration in the corrosion medium were studied. —(21.2.2).

Mechanical properties at room temperature of four cermets of tungsten carbide with cobalt binder. A. E. Johnson. N.A.C.A. Technical Note 3309 (*December* 1954).

Room-temperature stress-strain curves are presented for compression, tension, and shear loadings on four compositions of tungsten carbide with cobalt binder. Values of modulus of elasticity, modulus of rigidity, Poisson's ratio in the elastic region, ultimate strength, density, and hardness for the four materials are tabulated.—(21.3.1).

Tensile and compressive stress-strain properties of some high-strength sheet alloys at elevated temperatures. P. J. Hughes, J. E. Inge and S. B. Prosser. N.A.C.A. Technical Note 3315 (*November* 1954).

Results of tensile and compressive stress-strain tests at temperatures up to 1,200° F. are presented for S.A.E. 4340, Hy-Tuf, Stainless W, and Inconel X sheet materials which had ultimate tensile strengths at room temperature in the 170 to 220 k.s.i. range. Representative tensile and compressive stress-strain curves are given for each material at the test temperatures. Secant and tangent moduli, obtained from the compressive data, are included.—(21.2.1).

Corrosion of metals of construction by alternate exposure to liquid and gaseous fluorine. R. M. Gundzik and C. E. Feiler. N.A.C.A. Technical Note 3333 (*December* 1954.

The corrosion of 3S-0 and 52S-0 aluminium, AISI 347 and 321 stainless steels, "A" nickel, and low-leaded brass by alternate exposure to liquid and gaseous fluorine has been determined for periods of up to 3½ months and was found to be negligible.—(21.2.2 × 21.2.3).

MATHEMATICS

*The solution of linear simultaneous equations by matrix iteration. J. Guest. A.R.L. Report SM.*225 (*September* 1954).—(22.1).

MISSILES

*Flight measurements of drag and base pressure of a fin-stabilized parabolic body of revolution (N.A.C.A. RM-*10) *at different Reynolds numbers and at Mach numbers from* 0·9 *to* 3·3. *H. H. Jackson, C. B. Rumsey and L. T. Chauvin. N.A.C.A. Technical Note* 3320 (*November* 1954).

Free-flight tests at supersonic speeds have been made to determine the Reynolds number effects on total drag and base drag of a fin-stabilised parabolic-arc body of revolution having a body fineness ratio of 12·2 and designated the N.A.C.A. RM-10 configuration. The Reynolds number

range of 14×10^6 to 210×10^6 was obtained by testing full-scale and half-scale models through the Mach number range from 0·9 to 3·3.—(25.2 × 1.2.3).

NAVIGATION

A terminal VHF omnirange marker. T. S. Wonnell and B. H. Boyle. C.A.A. Technical Development Report No. 252 (*November* 1954).

A terminal VHF omni-range marker has been developed at the Technical Development and Evaluation Center of the Civil Aeronautics Administration. The purpose of the marker is to provide an audible position indication to aircraft on designated TVOR approach radials. The operation of the marker requires no marker receiver, since the marker signals are received on the VOR navigation receiver simultaneously with the TVOR signals.—(26.0 × 11).

POWER PLANTS

See also THERMODYNAMICS: THERMODYNAMIC PROPERTIES

A method for measuring size of drops in high-velocity sprays. D. R. Warren and A. Sharpe. A.R.L. Mechanical Engineering Note 188 (*May* 1954).

Methods have been considered for measuring the size of drops in concentrated sprays at high velocities, such as those encountered in ram-jet and turbine combustion chambers. A raindrop sampling technique devised by the C.S.I.R.O. Radiophysics Section, Sydney, has been modified so that a magnesium oxide coated rod is fired rapidly through a small isolated portion of the spray.—(27.3 × 27.4).

*Fire detection studies in the Convair-*340 *power plant. L. A. Asadourian. C.A.A. Technical Development Report No.* 250 (*November* 1954).

Tests were conducted in an operating full-scale Convair-340 aeroplane nacelle to evaluate the fire-detector system currently used in operational aircraft of this type.—(27.2).

Mechanism of start and development of aircraft crash fires. I. I. Pinkel, G. M. Preston and G. J. Pesman. N.A.C.A. Report 1133 (1953).

Full-scale aircraft crashes were made to investigate the mechanism of the start and development of aircraft crash fires. This investigation revealed the characteristics of the ignition sources, the manner in which the combustibles spread, the mechanism of the union of the combustibles and ignition sources, and the pertinent factors governing the development of a crash fire as observed in this programme. This report was formerly RM E52F06.—(27.2.0 × 5.3).

Some measurements of noise from three solid-fuel rocket engines. L. W. Lassiter and R. H. Heitkotter. N.A.C.A. Technical Note 3316 (*December* 1954).

Results of a systematic experimental investigation of the sound field of one type of rocket engine are given. These comprise frequency spectra and overall pressure data as obtained along a semi-circle of 50-foot radius about the engine. Frequency spectra and overall pressure magnitudes are also given for a few isolated points in the sound fields of two other solid-fuel rockets.—(27.3).

PROPELLERS

See also AERODYNAMICS: FLUID DYNAMICS

Investigation of the aerodynamic characteristics of a model wing-propeller combination and of the wing and propeller separately at angles of attack up to 90°. *J. W. Draper and R. E. Kuhn. N.A.C.A. Technical Note* 3304 (*November* 1954).

Results are presented of a wind-tunnel investigation of the effects of slipstream for two large-diameter propellers on the aerodynamic characteristics of a wing model. The investigation covered angles of attack from −10° to 90° and thrust coefficients representing free-stream velocities from zero to the normal range of cruising flight.—(29.1 × 1.10.2.2).

An investigation of a wing-propeller configuration employing large-chord plain flaps and large-diameter propellers for low-speed flight and vertical take-off. R. E. Kuhn and J. W. Draper. N.A.C.A. Technical Note 3307 (*December* 1954).

An investigation of the effectiveness of a wing equipped with large-chord plain flaps and auxiliary vanes in rotating the effective thrust vector from two large-diameter propellers to a near vertical attitude for vertical take-off and low-speed flight has been conducted in the Langley 300 m.p.h. 7- by 10-foot tunnel.—(29.1 × 1.10.2.2 × 1.3.4).

STRUCTURES

LOADS

On force-deflection diagrams of airplane shock absorber struts. First, second and third partial reports. K. Schlaefke. N.A.C.A. Technical Memorandum 1373 (*November* 1954).

The behaviour of landing-gear shock struts is analysed, with various types of assumptions for the shock-strut characteristics. The effects of tyre springing are neglected. The first part compares the behaviour of struts with linear and quadratic damping. The second part considers struts with non-linear spring characteristics and linear or quadratic damping. The third part treats the oleo-pneumatic strut with air-compression springing without damping and with damping proportional to velocity.—(33.1.2).

Determination of the elastic constants of airplane tires. Boeckh. N.A.C.A. Technical Memorandum 1378 (*November* 1954).

Measurements were made of the distortion of four German aircraft tyres, from about 22 to 28 inches in diameter, at several vertical loadings. For each vertical loading measurements were made of the tyre distortion under several lateral, tangential and torsional loadings.—(33.1.2).

Contribution to the theory of tail-wheel shimmy. M. Melzer. N.A.C.A. Technical Memorandum 1380 (*December* 1954).

A basic theoretical and experimental investigation is made of the shimmy behaviour of a swivelling landing gear, the experimental tests being conducted with a small wheel mounted over a continuous belt. Effects of wheel loading, rolling velocity, rearward position of the wheel with respect to the swivel axis, tyre elasticity, and torsional flexibility of the fuselage are investigated both experimentally and theoretically.—(33.1.2).

Full-scale measurements of impact loads on a large flying boat. Part I. Description of apparatus and instrument installation. J. W. McIvor. A.R.C. Current Paper 182 (*March* 1950, *published* 1954).

The variations with time of the total force and the distribution of water pressures on the hull bottom of a flying boat are related to the horizontal velocity, vertical velocity and keel attitude relative to the water during the impact. Methods are described for obtaining, in a form suitable for easy analysis of results, records of these variables, in order to verify impact theories.—(33.1.2).

A study of the application of power-spectral methods of generalized harmonic analysis to gust loads on airplanes. H. Press and B. Mazelsky. N.A.C.A. Report 1172 (1954).

The applicability of some results from the theory of generalised harmonic analysis to the analysis of gust loads on aeroplanes in continuous rough air is examined. The input and output relations in terms of power spectrums are used to relate the standard deviation (root mean square) of loads in continuous rough air to the gust response characteristics of the aeroplane and the spectral characteristics of atmospheric turbulence.—(33.1.2).

The influence of wheel spin-up on landing-gear impact. W. Flugge and C. W. Coale. N.A.C.A. Technical Note 3217 (*October* 1954).

This report deals with the influence of wheel drag on landing-gear performance. The differential equations are developed and solved by numerical integration and by an analytical method. Emphasis is placed on dropping influences of minor importance to simplify the computations. Consideration is also given to the influence of offset wheels, inclined shock struts, and friction due to ovalisation of the shock-strut cylinder under bending loads.—(33.1.2).

Some measurements and power spectra of runway roughness. J. H. Walls, J. C. Houbolt and H. Press. N.A.C.A. Technical Note 3305 (*November* 1954).

Measurements of actual runway roughness obtained by a profile-survey method (engineer's level) are presented. Data were obtained from a survey of a relatively rough runway

and a smooth runway. The results of this study are presented as roughness profiles of the runways surveyed and in the form of power spectra.—(33.1.2).

Some measurements of atmospheric turbulence obtained from flow-direction vanes mounted on an airplane. R. G. Chilton. N.A.C.A. Technical Note 3313 (*November* 1954).

The power spectrum of high-frequency turbulence in the atmosphere was calculated from measurements made in flight. Flow-direction vanes were used to measure vertical and horizontal components of gust velocity relative to the aeroplane and normal to the flight direction. The power spectral densities of the two components were, for practical purposes, equal.—(33.1.1).

THEORY AND ANALYSIS

Electrical analogies for stiffened shells with flexible rings. R. H. MacNeal. N.A.C.A. Technical Note 3280 (*December* 1954).

First, an electrical analogy is developed for a circular shell with a straight axis and variable radius and extended to non-circular cylinders. Then an electrical analogy is derived for rings with variable radii of curvature; a simplified circuit for circular rings is also presented. The simplifications that occur when the rings are assumed to be rigid are discussed. Finally, results are given for two sample problems solved on an analog computer. The second problem concerns a cantilever conical shell and illustrates the manner in which the shell and ring circuits are interconnected.—(33.2.4.11).

Investigation of static strength and creep behavior of an aluminum-alloy multiweb box beam at elevated temperatures. E. E. Mathauser. N.A.C.A. Technical Note 3310 (*November* 1954).

Methods that were used to predict failure stresses in the static-strength tests were in good agreement with the experimental results. Creep deflections and creep lifetimes are presented for beams subjected to constant load and various heating conditions. Lifetime is satisfactorily predicted from material stress-rupture data when tensile failure occurs at both constant and varying temperatures.—(33.2.4.1.9).

TESTING

The M.A.E.E. recording accelerometer. D. M. Ridland and R. Parker. A.R.C. Current Paper 177 (*September* 1952, *published* 1954).

The M.A.E.E. recording accelerometer is basically the accelerometer unit of a desynn accelerometer, adapted to make a continuous and immediate presentation of accurate, calibrated accelerations on a half second time base. The recording medium is metallised paper, having a speed of half an inch per second, and the instrument can be operated continuously for twenty minutes on one loading. It can record with full scale deflections, from 1 to 10 *g* when the natural frequencies will be about 7 and 22 c.p.s. respectively. —(33.3.1).

Recent strain gauge developments at A.R.L. E. S. Moody and D. R. Denehy. A.R.L. Structures and Materials Note 211 (*August* 1954).

Two instruments, for the measurement of direct load and torque respectively, are described. Both instruments have unbonded strain gauges as the sensitive elements. The application of recently developed "potting" techniques to strain gauge wiring is also described.—(33.3.1).

THERMODYNAMICS

THERMODYNAMIC PROPERTIES

Contribution a l'étude du mécanisme de la réaction lente entre l'oxygène et une substance organique gazeuse. M. Niclause. Publications Scientifiques du Ministère de l'Air, France. No. 292 (1954).—(34.1.1).

*Diffusion flames in the laboratory. J. Barr. AGARD Memorandum. AG*11/*M*7 (*May* 1954).—(34.1.1).

*Some aspects of combustion of liquid fuel. C. C. Graves and M. Gerstein. Agard Memorandum AG*16/*M*10 (*May* 1954).

The combustion of liquid sprays is considered in terms of such individual processes as fuel spray spreading and evaporation. In addition, the efficiency of combustion of liquid fuels in turbo-jet combustors as affected by fuel volatility, spray characteristics, and the burning rate of single drops is treated.—(34.1.1).

*The mechanism of carbon formation. G. Porter. Agard Memorandum AG*13/*M*9 (*May* 1954).

This paper gives a more detailed account of the theory of carbon formation in combustion processes which has been briefly stated elsewhere. Other theories are critically reviewed and a summary is given of the experimental data relating to carbon formation which has been obtained by the flash photolysis technique.—(34.1.1).

Photographic investigation of combustion in a two-dimensional transparent rocket engine. D. R. Bellman, J. C. Humphrey and T. Male. N.A.C.A. Report 1134 (1953).

Motion pictures at camera speeds up to 3,000 frames per second were taken of the combustion of liquid oxygen and gasoline in a 100-pound thrust rocket engine. The effect of seven methods of propellant injection on the uniformity of combustion was investigated.—(34.1.1 × 27.3).

Vaporization rates and drag coefficients for iso-octane sprays in turbulent air streams. R. D. Ingebo. N.A.C.A. Technical Note 3265 (*October* 1954).

Drop-size distribution and drop-velocity data were obtained for iso-octane sprays in turbulent air streams using a droplet camera developed at the N.A.C.A. Lewis laboratory. Experimental spray vaporisation rates, based on the mean diameter, correlated single-droplet vaporisation rates. An empirical expression was derived for iso-octane droplet drag coefficients.—(34.1).

HEAT TRANSFER

Heat transfer by free convection from horizontal cylinders in diatomic gases. R. Hermann. N.A.C.A. Technical Memorandum 1366 (*November* 1954).

Heat transfer by free convection from horizontal cylinders in diatomic gases is investigated theoretically and experimentally. A review and discussion of previous investigations of free-convection heat transfer from horizontal cylinders is included.—(34.3.2).

Theoretical and experimental investigation of heat transfer by laminar natural convection between parallel plates. A. F. Lietzke. N.A.C.A. Technical Note 3328 (*December* 1954).

For the experimental work, parallel walls (one heated uniformly and the other cooled uniformly) were simulated by an annulus with an inner-to-outer diameter ratio near 1. The theoretical results are presented in equations for the velocity and temperature profiles and the ratio of actual temperature drop across the fluid to temperature drop for pure conduction.—(34.3.2).

THERMODYNAMIC THEORY

Equilibres physico-chimiques et données thermodynamiques des melanges gazeux aux temperatures elevées. G. Ribaud and N. Manson. Publications Scientifiques et Techniques du Ministère de L'Air. No. 294 (1954).—(34.4).

Kinetic treatment of the nucleation in supersaturated vapors. R. Becker and W. Doring. N.A.C.A. Technical Memorandum. 1374 (*September* 1954).

The equations of the individual processes of self-nucleation are utilised through an electrical analogy to obtain the nucleation frequency. With the nucleation frequencies of crystals and spheres the Ostwald law of stages is reviewed and modified. In the final section the general resistance image is discussed and mention is made of the relation of the electrical network and Volmers formula.—(34.4).

Transverse oscillations in a cylindrical combustion chamber. F. K. Moore and S. H. Maslen. N.A.C.A. Technical Note 3152 (*October* 1954).

Transverse oscillations in combustion chambers are studied. With an axial temperature gradient considered, the modes of weak oscillation are described for low Mach number. Amplification by coupling with vigorous combustion in the flame-holder wake is assumed. The amplification depends on flame-holder and centrebody diameters and on time-lag effects. The nature of finite transverse periodic waves is also analysed.—(34.4).